Gödel's Incompleteness Theorems

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Abstract

Gdel's two incompleteness theorems [2] are formalised, following a careful presentation by Świerczkowski [4], in the theory of hereditarily finite sets. This represents the first ever machine-assisted proof of the second incompleteness theorem. Compared with traditional formalisations using Peano arithmetic [1], coding is simpler, with no need to formalise the notion of multiplication (let alone prime number) in the formalised calculus upon which the theorem is based. (The definition of addition for the HF sets follows Kirby [3].) However, other technical problems had to be solved in order to complete the argument.

Contents

Bibliography

[1] G. S. Boolos. *The Logic of Provability*. Cambridge University Press, 1993.

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[3] L. Kirby. Addition and multiplication of sets. *Mathematical Logic Quarterly*, 53(1):52–65, 2007.

[4] S. Świerczkowski. Finite sets and Gödel's incompleteness theorems. *Dissertationes Mathematicae*, 422:1–58, 2003. http://journals.impan.gov.pl/dm/Inf/422-0-1.html.