

A Meta-Model for the Isabelle API

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Abstract

We represent a theory *of* (a fragment of) Isabelle/HOL *in* Isabelle/HOL. The purpose of this exercise is to write packages for domain-specific specifications such as class models, B-machines, . . . , and generally speaking, any domain-specific languages whose abstract syntax can be defined by a HOL “datatype”. On this basis, the Isabelle code-generator can then be used to generate code for global context transformations as well as tactic code.

Consequently the package is geared towards parsing, printing and code-generation to the Isabelle API. It is at the moment not sufficiently rich for doing meta theory on Isabelle itself. Extensions in this direction are possible though.

Moreover, the chosen fragment is fairly rudimentary. However it should be easily adapted to one’s needs if a package is written on top of it. The supported API contains types, terms, transformation of global context like definitions and data-type declarations as well as infrastructure for Isar-setups.

This theory is drawn from the Featherweight OCL[1] project where it is used to construct a package for object-oriented data-type theories generated from UML class diagrams. The Featherweight OCL, for example, allows for both the direct execution of compiled tactic code by the Isabelle API as well as the generation of `.thy`-files for debugging purposes.

Gained experience from this project shows that the compiled code is sufficiently efficient for practical purposes while being based on a formal *model* on which properties of the package can be proven such as termination of certain transformations, correctness, etc.

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Part I.

A Meta-Model for the Isabelle API

1. Initialization

```
theory Init
  imports isabelle-home/src/HOL/Isabelle-Main0
begin
```

1.1. Optimization on the String Datatype

The following types will allow to delay all concatenations on *integer list*, until we reach the end. As optimization, we also consider the use of *String.literal* besides *integer list*.

```
type-notation natural (<nat>)
definition Succ x = x + 1
```

```
datatype stringbase = ST String.literal
  | ST' integer list
```

```
datatype abr-string =
  SS-base stringbase
  | String-concatWith abr-string abr-string list
```

```
syntax -string1 :: - ⇒ abr-string (<<(-)>>)
translations <x> ⇒ CONST SS-base (CONST ST x)
```

```
syntax -string3 :: - ⇒ abr-string (<<<(-)>>>)
translations <<x>> ⇒ CONST SS-base (CONST ST' x)
```

```
syntax -integer1 :: - ⇒ abr-string (<°(-)°>)
translations °x° ⇒ CONST SS-base (CONST ST' ((CONST Cons) x (CONST Nil)))
```

```
type-notation abr-string (<string>)
```

1.2. Basic Extension of the Standard Library

1.2.1. Polymorphic Cartouches

We generalize the construction of cartouches for them to be used “polymorphically”, however the type inference is not automatic: types of all cartouche expressions will need to be specified earlier before their use (we will however provide a default type).

$\langle ML \rangle$

This is the special command which sets the type of subsequent cartouches. Note: here the given type is currently parsed as a string, one should extend it to be a truly “typed” type...

```
declare[[cartouche-type = abr-string]]
```

1.2.2. Operations on List

```
datatype ('a, 'b) nsplit = Nsplit-text 'a
                        | Nsplit-sep 'b
```

```
locale L
```

```
begin
```

```
definition map where map f l = rev (foldl ( $\lambda x. f x \# l$ ) [] l)
```

```
definition flatten l = foldl ( $\lambda acc l. foldl (\lambda acc x. x \# acc) acc (rev l)$ ) [] (rev l)
```

```
definition mapi f l = rev (fst (foldl ( $\lambda(l,cpt) x. (f cpt x \# l, Succ cpt)$ ) ([], 0::nat) l))
```

```
definition iter f = foldl ( $\lambda-. f$ ) ()
```

```
definition maps f x = L.flatten (L.map f x)
```

```
definition append where append a b = L.flatten [a, b]
```

```
definition filter where filter f l = rev (foldl ( $\lambda x. if f x then x \# l else l$ ) [] l)
```

```
definition rev-map f = foldl ( $\lambda x. f x \# l$ ) []
```

```
definition mapM f l accu =
```

```
(let (l, accu) = List.fold ( $\lambda x (l, accu). let (x, accu) = f x accu in (x \# l, accu)$ ) l ([], accu) in
  (rev l, accu))
```

```
definition assoc x1 l = List.fold ( $\lambda(x2, v). \lambda None \Rightarrow if x1 = x2 then Some v else None | x \Rightarrow x$ ) l None
```

```
definition split where split l = (L.map fst l, L.map snd l)
```

```
definition upto where upto i j =
```

```
(let to-i =  $\lambda n. int-of-integer (integer-of-natural n)$  in
  L.map (natural-of-integer o integer-of-int) (List.upto (to-i i) (to-i j)))
```

```
definition split-at f l =
```

```
(let f =  $\lambda x. \neg f x$  in
  (takeWhile f l, case dropWhile f l of []  $\Rightarrow$  (None, []) | x # xs  $\Rightarrow$  (Some x, xs)))
```

```
definition take where take reverse lg l = reverse (snd (L.split (takeWhile ( $\lambda(n, -). n < lg$ )
  (enumerate 0 (reverse l)))))
```

```
definition take-last = take rev
```

```
definition take-first = take id
```

```
definition replace-gen f-res l c0 lby =
```

```
(let Nsplit-text =  $\lambda l lgen. if l = [] then lgen else Nsplit-text l \# lgen$  in
  case List.fold
```

```
( $\lambda c1 (l, lgen).
  if c0 c1 then
    (lby, Nsplit-sep c1 \# Nsplit-text l lgen)
  else
    (c1 \# l, lgen))
  (rev l)
  ([], [])$ 
```

```
of (l, lgen)  $\Rightarrow f-res (Nsplit-text l lgen)$ )
```

```
definition nsplit-f l c0 = replace-gen id l c0 []
```

```
definition replace = replace-gen (L.flatten o L.map ( $\lambda Nsplit-text l \Rightarrow l | - \Rightarrow []$ ))
```

```

fun map-find-aux where
  map-find-aux accu f l = (λ [] ⇒ List.rev accu
    | x # xs ⇒ (case f x of Some x ⇒ List.fold Cons accu (x # xs)
    | None ⇒ map-find-aux (x # accu) f xs)) l
definition map-find = map-find-aux []
end
notation L.append (infixr ‹@@@@› 65)

```

lemmas [code] =

```

— def
L.map-def
L.flatten-def
L.mapi-def
L.iter-def
L.maps-def
L.append-def
L.filter-def
L.rev-map-def
L.mapM-def
L.assoc-def
L.split-def
L.upto-def
L.split-at-def
L.take-def
L.take-last-def
L.take-first-def
L.replace-gen-def
L.nsplit-f-def
L.replace-def
L.map-find-def

— fun
L.map-find-aux.simps

```

1.2.3. Operations on Char

definition *ascii-of-literal* (‹INT›) **where**
ascii-of-literal = hd o *String.asciis-of-literal*

definition (*integer-escape* :: *integer*) = 0x09

definition *ST0* c = ‹‹[c]››

definition *ST0-base* c = *ST'* [c]

1.2.4. Operations on String (I)

notation *String.asciis-of-literal* (‹INTS›)

locale *S*

locale *String*
locale *String_{base}*

definition (in *S*) *flatten* = *String-concatWith* $\langle \rangle$

definition (in *String*) *flatten* *a b* = *S.flatten* [*a*, *b*]

notation *String.flatten* (**infixr** $\langle @@ \rangle$ 65)

definition (in *String*) *make n c* = $\ll L.map (\lambda -. c) (L.upto\ 1\ n) \gg$

definition (in *String_{base}*) *map-gen replace g* = $(\lambda\ ST\ s \Rightarrow replace\ \langle \rangle\ (Some\ s)\ \langle \rangle$
 $\quad | ST'\ s \Rightarrow S.flatten\ (L.map\ g\ s))$

fun (in *String*) *map-gen where*

map-gen replace g e =

$(\lambda\ SS-base\ s \Rightarrow String_{base}.map-gen\ replace\ g\ s$

$\quad | String-concatWith\ abr\ l \Rightarrow String-concatWith\ (map-gen\ replace\ g\ abr)\ (List.map\ (map-gen\ replace\ g)\ l))\ e$

definition (in *String*) *foldl-one f accu* = *foldl f accu o INTS*

definition (in *String_{base}*) *foldl where foldl f accu* = $(\lambda\ ST\ s \Rightarrow String.foldl-one\ f\ accu\ s$
 $\quad | ST'\ s \Rightarrow List.foldl\ f\ accu\ s)$

fun (in *String*) *foldl where*

foldl f accu e =

$(\lambda\ SS-base\ s \Rightarrow String_{base}.foldl\ f\ accu\ s$

$\quad | String-concatWith\ abr\ l \Rightarrow$

$\quad (case\ l\ of\ [] \Rightarrow accu$

$\quad \quad | x\ \#\ xs \Rightarrow List.foldl\ (\lambda accu.\ foldl\ f\ (foldl\ f\ accu\ abr))\ (foldl\ f\ accu\ x)\ xs))\ e$

definition (in *S*) *replace-integers f s1 s s2* =

s1 @@ (case s of None $\Rightarrow \langle \rangle$ | Some s $\Rightarrow flatten\ (L.map\ f\ (INTS\ s))$) @@ s2

definition (in *String*) *map where map f* = *map-gen (S.replace-integers ($\lambda c.\ ^\circ f\ c^\circ$)) ($\lambda x.\ ^\circ f\ x^\circ$)*

definition (in *String*) *replace-integers f* = *map-gen (S.replace-integers ($\lambda c.\ f\ c$)) f*

definition (in *String*) *all f* = *foldl ($\lambda b\ s.\ b\ \&\ f\ s$) True*

definition (in *String*) *length where length* = *foldl ($\lambda n\ -. Suc\ n$) 0*

definition (in *String*) *to-list s* = *rev (foldl ($\lambda l\ c.\ c\ \#\ l$) [] s)*

definition (in *String_{base}*) *to-list* = $(\lambda\ ST\ s \Rightarrow INTS\ s\ | ST'\ l \Rightarrow l)$

definition (in *String*) *meta-of-logic* = *String.literal-of-ascii o to-list*

definition (in *String*) *to-String_{base}* = $(\lambda\ SS-base\ s \Rightarrow s\ | s \Rightarrow ST'\ (to-list\ s))$

definition (in *String_{base}*) *to-String* = *SS-base*

definition (in *String_{base}*) *is-empty* = $(\lambda\ ST\ s \Rightarrow s = STR\ \text{''''}$
 $\quad | ST'\ s \Rightarrow s = [])$

fun (in *String*) *is-empty where*

is-empty e = $(\lambda\ SS-base\ s \Rightarrow String_{base}.is-empty\ s\ | String-concatWith\ -\ l \Rightarrow list-all\ is-empty\ l)\ e$

definition (in *String*) *equal s1 s2* = $(to-list\ s1 = to-list\ s2)$

notation *String.equal* (**infixl** $\langle \triangle \rangle$ 50)

definition (in *String*) *assoc x l* = *L.assoc (to-list x) (L.map (map-prod String_{base}.to-list id) l)*

definition (in *String*) *member l x* = *List.member (L.map String_{base}.to-list l) (to-list x)*

definition (in *String_{base}*) *flatten l* = *String.to-String_{base} (S.flatten (L.map to-String l))*

lemmas [*code*] =

— def

S.flatten-def

String.flatten-def

String.make-def
String_base.map-gen-def
String.foldl-one-def
String_base.foldl-def
S.replace-integers-def
String.map-def
String.replace-integers-def
String.all-def
String.length-def
String.to-list-def
String_base.to-list-def
String.meta-of-logic-def
String.to-String_base-def
String_base.to-String-def
String_base.is-empty-def
String.equal-def
String.assoc-def
String.member-def
String_base.flatten-def

— fun

String.map-gen.simps
String.foldl.simps
String.is-empty.simps

1.2.5. Operations on String (II)

definition *wildcard* = ⟨-⟩

context *String*

begin

definition *lowercase* = map (λn. if n < 97 then n + 32 else n)

definition *uppercase* = map (λn. if n < 97 then n else n - 32)

definition *to-bold-number* = replace-integers (λn. [⟨0⟩, ⟨1⟩, ⟨2⟩, ⟨3⟩, ⟨4⟩, ⟨5⟩, ⟨6⟩, ⟨7⟩, ⟨8⟩, ⟨9⟩] ! nat-of-integer (n - 48))

fun *nat-to-digit10-aux* **where**

nat-to-digit10-aux l (n :: Nat.nat) = (if n < 10 then n # l else nat-to-digit10-aux (n mod 10 # l) (n div 10))

definition *nat-to-digit10* n =

(let *nat-raw-to-str* = L.map (integer-of-nat o (+) 0x30) in

⟨⟨nat-raw-to-str (nat-to-digit10-aux [] n)⟩⟩)

definition *natural-to-digit10* = nat-to-digit10 o nat-of-natural

declare[[*cartouche-type* = *String.literal*]]

definition *integer-to-digit16* =

(let *f* = nth (INTS ⟨0123456789ABCDEF⟩) o nat-of-integer in

λn ⇒ ⟨⟨[f (n div 16), f (n mod 16)]⟩⟩)

end

```

lemmas [code] =
  — def
  String.lowercase-def
  String.uppercase-def
  String.to-bold-number-def
  String.nat-to-digit10-def
  String.natural-to-digit10-def
  String.integer-to-digit16-def

  — fun
  String.nat-to-digit10-aux.simps

definition add-0 n =
  (let n = nat-of-integer n in
  S.flatten (L.map (λ-. ⟨0⟩) (upt 0 (if n < 10 then 2 else if n < 100 then 1 else 0))))
  @@ String.nat-to-digit10 n

declare[[cartouche-type = String.literal]]

definition is-letter =
  (let int-A = INT ⟨A⟩; int-Z = INT ⟨Z⟩; int-a = INT ⟨a⟩; int-z = INT ⟨z⟩ in
  (λn. n ≥ int-A & n ≤ int-Z | n ≥ int-a & n ≤ int-z))

definition is-digit =
  (let int-0 = INT ⟨0⟩; int-9 = INT ⟨9⟩ in
  (λn. n ≥ int-0 & n ≤ int-9))

definition is-special = List.member (INTS ⟨ <> ^-=-./(){}⟩)

context String
begin
definition base255 = replace-integers (λc. if is-letter c then °c° else add-0 c)
declare[[cartouche-type = abr-string]]
definition isub =
  replace-integers (let is-und = List.member (INTS (STR "-")) in
  (λc. if is-letter c | is-digit c | is-und c then ⟨, @@ °c° else add-0 c))
definition isup s = ⟨--> @@ s
end
lemmas [code] =
  — def
  String.base255-def
  String.isub-def
  String.isup-def

declare[[cartouche-type = abr-string]]

definition text-of-str str =
  (let s = ⟨c⟩
  ; ap = ⟨ # ⟩ in
  S.flatten [ ⟨(let ⟨, s, ⟨ = char-of :: nat ⇒ char in ⟩
  , String.replace-integers (λc.
  if is-letter c then

```

```

      S.flatten [⟨CHR "°,c°,⟨"⟩,ap]
    else
      S.flatten [s, ⟨ ⟩, add-0 c, ap]
  str
  , ⟨[]⟩])

```

definition $\langle \text{text2-of-str} = \text{String.replace-integers } (\lambda c. S.\text{flatten } [\langle \backslash \rangle, \langle < \rangle, \text{°}c\text{°}, \langle > \rangle]) \rangle$

definition $\text{textstr-of-str f-flatten f-integer f-str str} =$
 $(\text{let str0} = \text{String.to-list str}$
 $; f\text{-letter} = \lambda c. \text{is-letter } c \mid \text{is-digit } c \mid \text{is-special } c$
 $; s = \langle c \rangle$
 $; f\text{-text} = \lambda \text{Nsplits-text } l \Rightarrow S.\text{flatten } [f\text{-str } (S.\text{flatten } [\langle \text{STR "°,}\langle\langle l \rangle\rangle, \langle " \rangle])]$
 $\mid \text{Nsplits-sep } c \Rightarrow S.\text{flatten } [f\text{-integer } c]$
 $; str = \text{case } L.\text{nsplit-f str0 } (\text{Not } o f\text{-letter}) \text{ of}$
 $[] \Rightarrow S.\text{flatten } [f\text{-str } \langle \text{STR ""} \rangle]$
 $\mid [x] \Rightarrow f\text{-text } x$
 $\mid l \Rightarrow S.\text{flatten } (L.\text{map } (\lambda x. \langle \rangle \text{@@ } f\text{-text } x \text{@@} \langle \rangle \# \rangle) l) \text{@@} \langle [] \rangle \text{ in}$
 $\text{if list-all } f\text{-letter str0 \text{ then}}$
 str
 else
 $f\text{-flatten } (S.\text{flatten } [\langle \rangle, str, \langle \rangle]))$

definition $\langle \text{escape-sml} = \text{String.replace-integers } (\lambda n. \text{if } n = 0x22 \text{ then } \langle \backslash \rangle \text{ else } \text{°}n\text{°}) \rangle$

definition $\text{mk-constr-name name} = (\lambda x. S.\text{flatten } [\text{String.isub name}, \langle \text{-} \rangle, \text{String.isub } x])$

definition $\text{mk-dot s1 s2} = S.\text{flatten } [\langle \text{.} \rangle, s1, s2]$

definition $\text{mk-dot-par-gen dot l-s} = S.\text{flatten } [\text{dot}, \langle \rangle, \text{case } l\text{-s of } [] \Rightarrow \langle \rangle \mid x \# xs \Rightarrow S.\text{flatten } [x, S.\text{flatten } (L.\text{map } (\lambda s. \langle \rangle \text{@@} s) xs)], \langle \rangle]$

definition $\text{mk-dot-par dot s} = \text{mk-dot-par-gen dot } [s]$

definition $\text{mk-dot-comment s1 s2 s3} = \text{mk-dot } s1 (S.\text{flatten } [s2, \langle / * \rangle, s3, \langle * / \rangle])$

definition $\text{hol-definition } s = S.\text{flatten } [s, \langle \text{-def} \rangle]$

definition $\text{hol-split } s = S.\text{flatten } [s, \langle \text{.split} \rangle]$

end

2. Defining Meta-Models

2.1. (Pure) Term Meta-Model aka. AST definition of (Pure) Term

```
theory Meta-Pure
imports ../Init
begin
```

2.1.1. Type Definition

```
type-synonym indexname = string × nat
type-synonym class = string
type-synonym sort = class list
datatype typ =
  Type string typ list |
  TFree string sort |
  TVar indexname sort
datatype term =
  Const string typ |
  Free string typ |
  Var indexname typ |
  Bound nat |
  Abs string typ term |
  App term term (infixl <$> 200)
```

2.1.2. Operations of Fold, Map, ..., on the Meta-Model

```
fun map-Const where
  map-Const f expr = (λ Const s ty ⇒ Const (f s ty) ty
    | Free s ty ⇒ Free s ty
    | Var i ty ⇒ Var i ty
    | Bound n ⇒ Bound n
    | Abs s ty term ⇒ Abs s ty (map-Const f term)
    | App term1 term2 ⇒ App (map-Const f term1)
      (map-Const f term2))
  expr
```

```
fun fold-Const where
  fold-Const f accu expr = (λ Const s - ⇒ f accu s
    | Abs - - term ⇒ fold-Const f accu term
    | App term1 term2 ⇒ fold-Const f (fold-Const f accu term1) term2
    | - ⇒ accu)
```

expr

```
fun fold-Free where
  fold-Free f accu expr = (λ Free s - ⇒ f accu s
    | Abs - - term ⇒ fold-Free f accu term
    | App term1 term2 ⇒ fold-Free f (fold-Free f accu term1) term2
    | - ⇒ accu)
  expr

end
```

2.2. SML Meta-Model aka. AST definition of SML

```
theory Meta-SML
imports ../Init
begin
```

2.2.1. Type Definition

The following datatypes beginning with `semi__` represent semi-concrete syntax, deliberately not minimal abstract syntax like (Pure) Term, this is for example to facilitate the pretty-printing process, or for manipulating recursively data-structures through an abstract and typed API.

```
datatype semi--val-fun = Sval
  | Sfun
```

```
datatype semi--term' = SML-string string
  | SML-rewrite semi--val-fun semi--term' — left string — symb rewriting
semi--term' — right
  | SML-basic string list
  | SML-binop semi--term' string semi--term'
  | SML-annot semi--term' string — type
  | SML-function (semi--term' — pattern × semi--term' — to return) list
  | SML-apply semi--term' semi--term' list
  | SML-paren string — left string — right semi--term'
  | SML-let-open string semi--term'
```

2.2.2. Extending the Meta-Model

```
locale SML
begin
no-type-notation abr-string (⟨string⟩) definition string = SML-string
definition rewrite = SML-rewrite
definition basic = SML-basic
definition binop = SML-binop
definition annot = SML-annot
definition function = SML-function
definition apply = SML-apply
```

```

definition paren = SML-paren
definition let-open = SML-let-open

definition app s = apply (basic [s])
definition none = basic [⟨NONE⟩]
definition some s = app ⟨SOME⟩ [s]
definition option' f l = (case map-option f l of None ⇒ none | Some s ⇒ some s)
definition option = option' id
definition parenthesis — mandatory parenthesis = paren ⟨(⟩ ⟨)⟩
definition binop-l s l = (case rev l of x # xs ⇒ List.fold (λx. binop x s) xs x)
definition list l = (case l of [] ⇒ basic [⟨[]⟩] | - ⇒ paren ⟨[⟩ ⟨]⟩ (binop-l ⟨,⟩ l))
definition list' f l = list (L.map f l)
definition pair e1 e2 = parenthesis (binop e1 ⟨,⟩ e2)
definition pair' f1 f2 = (λ (e1, e2) ⇒ parenthesis (binop (f1 e1) ⟨,⟩ (f2 e2)))
definition rewrite-val = rewrite Sval
definition rewrite-fun = rewrite Sfun
end

```

```

lemmas [code] =
  — def
  SML.string-def
  SML.rewrite-def
  SML.basic-def
  SML.binop-def
  SML.annot-def
  SML.function-def
  SML.apply-def
  SML.paren-def
  SML.let-open-def
  SML.app-def
  SML.none-def
  SML.some-def
  SML.option'-def
  SML.option-def
  SML.parenthesis-def
  SML.binop-l-def
  SML.list-def
  SML.list'-def
  SML.pair-def
  SML.pair'-def
  SML.rewrite-val-def
  SML.rewrite-fun-def

```

end

2.3. Isabelle Meta-Model aka. AST definition of Isabelle

```

theory Meta-Isabelle
imports Meta-Pure

```

begin

2.3.1. Type Definition

The following datatypes beginning with `semi__` represent semi-concrete syntax, deliberately not minimal abstract syntax like (Pure) Term, this is for example to facilitate the pretty-printing process, or for manipulating recursively data-structures through an abstract and typed API.

```
datatype semi--typ = Typ-apply semi--typ semi--typ list
  | Typ-apply-bin string — binop semi--typ semi--typ
  | Typ-apply-paren string — left string — right semi--typ
  | Typ-base string
```

```
datatype datatype = Datatype string — name
  (string — name × semi--typ list — arguments) list — constructors
```

```
datatype type-synonym = Type-synonym string — name
  string list — parametric variables
  semi--typ — content
```

```
datatype semi--term = Term-rewrite semi--term — left string — symb rewriting semi--term
— right
```

```
  | Term-basic string list
  | Term-annot semi--term semi--typ
  | Term-bind string — symbol semi--term — arg semi--term
  | Term-fun-case semi--term — value option — none: function (semi--term —
pattern × semi--term — to return) list
  | Term-apply semi--term semi--term list
  | Term-paren string — left string — right semi--term
  | Term-if-then-else semi--term semi--term semi--term
  | Term-term string list — simulate a pre-initialized context (de bruijn variables
under "lam")
  term — usual continuation of inner syntax term
```

```
datatype type-notation = Type-notation string — name
  string — content
```

```
datatype instantiation = Instantiation string — name
  string — name in definition
  semi--term
```

```
datatype overloading = Overloading string — name consts semi--term
  string — name def semi--term — content
```

```
datatype consts = Consts string — name
  semi--typ
  string — expression in 'post' mixfix
```

datatype *definition* = *Definition semi-term*
 | *Definition-where1 string* — name *semi-term* — syntax extension \times *nat* —
 priority *semi-term*
 | *Definition-where2 string* — name *semi-term* — syntax extension *semi-term*

datatype *semi-thm-attribute* = *Thm-thm string* — represents a single thm
 | *Thm-thms string* — represents several thms
 | *Thm-THEN semi-thm-attribute semi-thm-attribute*
 | *Thm-simplified semi-thm-attribute semi-thm-attribute*
 | *Thm-symmetric semi-thm-attribute*
 | *Thm-where semi-thm-attribute (string \times semi-term) list*
 | *Thm-of semi-thm-attribute semi-term list*
 | *Thm-OF semi-thm-attribute semi-thm-attribute*

datatype *semi-thm* = *Thms-single semi-thm-attribute*
 | *Thms-mult semi-thm-attribute*

type-synonym *semi-thm-l* = *semi-thm list*

datatype *lemmas* = *Lemmas-simp-thm bool* — True : simp
 string — name
 semi-thm-attribute list
 | *Lemmas-simp-thms string* — name
 string — thms list

datatype *semi-method-simp* = *Method-simp-only semi-thm-l*
 | *Method-simp-add-del-split semi-thm-l* — add *semi-thm-l* — del *semi-thm-l*
 — split

datatype *semi-method* = *Method-rule semi-thm-attribute option*
 | *Method-drule semi-thm-attribute*
 | *Method-erule semi-thm-attribute*
 | *Method-intro semi-thm-attribute list*
 | *Method-elim semi-thm-attribute*
 | *Method-subst bool* — asm
 string — nat list — pos
 semi-thm-attribute
 | *Method-insert semi-thm-l*
 | *Method-plus semi-method list*
 | *Method-option semi-method list*
 | *Method-or semi-method list*
 | *Method-one semi-method-simp*
 | *Method-all semi-method-simp*
 | *Method-auto-simp-add-split semi-thm-l string list*
 | *Method-rename-tac string list*
 | *Method-case-tac semi-term*
 | *Method-blast nat option*
 | *Method-clarify*

| *Method-metis string list* — e.g. *no-types (override-type-encls)*
semi--thm-attribute list

datatype *semi--command-final* = *Command-done*
| *Command-by semi--method list*
| *Command-sorry*

datatype *semi--command-state* = *Command-apply-end semi--method list* — **apply-end** (...
...)

datatype *semi--command-proof* = *Command-apply semi--method list* — **apply** (... , ...)
| *Command-using semi--thm-l* — **using** ...
| *Command-unfolding semi--thm-l* — **unfolding** ...
| *Command-let semi--term* — name *semi--term*
| *Command-have string* — name
bool — true: add [*simp*]
semi--term
semi--command-final
| *Command-fix-let string list*
(*semi--term* — name × *semi--term*) *list* — let statements
(*semi--term list* — **show** ... ⇒ ...
× *semi--term list* — **when**) *option* — *None* ⇒
?*thesis*
semi--command-state list — **qed apply-end** ...

datatype *lemma* = *Lemma string* — name *semi--term list* — specification to prove
semi--method list list — tactics: **apply** (... , ...) **apply** ...
semi--command-final
| *Lemma-assumes string* — name
(*string* — name × *bool* — true: add [*simp*] × *semi--term*) *list* —
specification to prove (assms)
semi--term — specification to prove (conclusion)
semi--command-proof list
semi--command-final

datatype *axiomatization* = *Axiomatization string* — name
semi--term

datatype *section* = *Section nat* — nesting level
string — content

datatype *text* = *Text string*

datatype *ML* = *SML semi--term'*

datatype *setup* = *Setup semi--term'*

datatype *thm* = *Thm semi--thm-attribute list*

```
datatype interpretation = Interpretation string — name
                           string — locale name
                           semi--term list — locale param
                           semi--command-final
```

```
datatype semi--theory = Theory-datatype datatype
  | Theory-type-synonym type-synonym
  | Theory-type-notation type-notation
  | Theory-instantiation instantiation
  | Theory-overloading overloading
  | Theory-consts consts
  | Theory-definition definition
  | Theory-lemmas lemmas
  | Theory-lemma lemma
  | Theory-axiomatization axiomatization
  | Theory-section section
  | Theory-text text
  | Theory-ML ML
  | Theory-setup setup
  | Theory-thm thm
  | Theory-interpretation interpretation
```

```
record semi--locale =
  HolThyLocale-name :: string
  HolThyLocale-header :: ( (semi--term — name × semi--typ — fix statement) list
    × (string — name × semi--term — assumes statement) option — None:
no assumes to generate) list
```

```
datatype semi--theories = Theories-one semi--theory
  | Theories-locale semi--locale semi--theory list — positioning comments can
occur before and after this group of commands list
```

2.3.2. Extending the Meta-Model

```
locale T
begin
definition thm = Thm-thm
definition thms = Thm-thms
definition THEN = Thm-THEN
definition simplified = Thm-simplified
definition symmetric = Thm-symmetric
definition where = Thm-where
definition of' = Thm-of
definition OF = Thm-OF
definition OF-l s l = List.fold ( $\lambda x \text{ acc. Thm-OF acc } x$ ) l s
definition simplified-l s l = List.fold ( $\lambda x \text{ acc. Thm-simplified acc } x$ ) l s
end
```

```
lemmas [code] =
```

— def
T.thm-def
T.thms-def
T.THEN-def
T.simplified-def
T.symmetric-def
T.where-def
T.of'-def
T.OF-def
T.OF-l-def
T.simplified-l-def

definition *Opt s = Typ-apply (Typ-base <option>) [Typ-base s]*

definition *Raw = Typ-base*

definition *Type-synonym' n = Type-synonym n []*

definition *Type-synonym'' n l f = Type-synonym n l (f l)*

definition *Term-annot' e s = Term-annot e (Typ-base s)*

definition *Term-lambdas s = Term-bind <λ> (Term-basic s)*

definition *Term-lambda x = Term-lambdas [x]*

definition *Term-lambdas0 = Term-bind <λ>*

definition *Term-lam x f = Term-lambdas0 (Term-basic [x]) (f x)*

definition *Term-some = Term-paren <|> <|>*

definition *Term-parenthesis* — mandatory parenthesis = *Term-paren <(>>*

definition *Term-warning-parenthesis* — optional parenthesis that can be removed but a warning will be raised = *Term-parenthesis*

definition *Term-pat b = Term-basic [⟨?⟩ @@ b]*

definition *Term-And x f = Term-bind <∧> (Term-basic [x]) (f x)*

definition *Term-exists x f = Term-bind <∃> (Term-basic [x]) (f x)*

definition *Term-binop = Term-rewrite*

definition *term-binop s l = (case rev l of x # xs ⇒ List.fold (λx. Term-binop x s) xs x)*

definition *term-binop' s l = (case rev l of x # xs ⇒ List.fold (λx. Term-parenthesis o Term-binop x s) xs x)*

definition *Term-set l = (case l of [] ⇒ Term-basic [⟨{ }⟩] | - ⇒ Term-paren <{ }> (term-binop <,> l))*

definition *Term-list l = (case l of [] ⇒ Term-basic [⟨[]⟩] | - ⇒ Term-paren <[]> (term-binop <,> l))*

definition *Term-list' f l = Term-list (L.map f l)*

definition *Term-pair e1 e2 = Term-parenthesis (Term-binop e1 <,> e2)*

definition *Term-pair' l = (case l of [] ⇒ Term-basic [⟨()⟩] | - ⇒ Term-paren <()> (term-binop <,> l))*

definition *<Term-string s = Term-basic [S.flatten [⟨,⟩, s, <⟩]]>*

definition *Term-applys0 e l = Term-parenthesis (Term-apply e (L.map Term-parenthesis l))*

definition *Term-applys e l = Term-applys0 (Term-parenthesis e) l*

definition *Term-app e = Term-applys0 (Term-basic [e])*

definition *Term-preunary e1 e2 = Term-apply e1 [e2]* — no parenthesis and separated with one space

definition *Term-postunary e1 e2 = Term-apply e1 [e2]* — no parenthesis and separated with one space

definition *Term-case = Term-fun-case o Some*

definition *Term-function* = *Term-fun-case None*
definition *Term-term'* = *Term-term* []
definition *Lemmas-simp* = *Lemmas-simp-thm True*
definition *Lemmas-nosimp* = *Lemmas-simp-thm False*
definition *Consts-value* = ⟨(-)⟩
definition *Consts-raw0 s l e o-arg* =
 Consts s l (String.replace-integers (λn. if n = 0x5F then ⟨'⟩ else °n°) e @@ (case o-arg of
 None ⇒ ⟨⟩
 | Some arg ⇒
 let ap = λs. ⟨'⟩ @@ s @@ ⟨'⟩ in
 ap (if arg = 0 then
 ⟨⟩
 else
 Consts-value @@ (S.flatten (L.map (λ-. ⟨'⟩ @@ Consts-value) (L.upto 2 arg))))))
definition *Ty-arrow* = *Typ-apply-bin ⟨⇒⟩*
definition *Ty-times* = *Typ-apply-bin ⟨×⟩*
definition *Ty-arrow' x* = *Ty-arrow x (Typ-base ⟨-⟩)*
definition *Ty-paren* = *Typ-apply-paren ⟨(⟩ ⟨'⟩)*
definition *Consts' s l e* = *Consts-raw0 s (Ty-arrow (Typ-base ⟨'α⟩) l) e None*
definition *Overloading' n ty* = *Overloading n (Term-annot (Term-basic [n]) ty)*

locale *M*
begin
definition *Method-simp-add-del l-a l-d* = *Method-simp-add-del-split l-a l-d* []
definition *Method-subst-l* = *Method-subst False*

definition *rule'* = *Method-rule None*
definition *rule* = *Method-rule o Some*
definition *drule* = *Method-drule*
definition *erule* = *Method-erule*
definition *intro* = *Method-intro*
definition *elim* = *Method-elim*
definition *subst-l0* = *Method-subst*
definition *subst-l* = *Method-subst-l*
definition *insert where insert* = *Method-insert o L.map Thms-single*
definition *plus where plus* = *Method-plus*
definition *option* = *Method-option*
definition *or where or* = *Method-or*
definition *meth-gen-simp* = *Method-simp-add-del* [] []
definition *meth-gen-simp-add2 l1 l2* = *Method-simp-add-del (L.flatten [L.map Thms-mult l1*
 , L.map (Thms-single o Thm-thm) l2])
 []
definition *meth-gen-simp-add-del l1 l2* = *Method-simp-add-del (L.map (Thms-single o Thm-thm)*
l1)
 (L.map (Thms-single o Thm-thm) l2)
definition *meth-gen-simp-add-del-split l1 l2 l3* = *Method-simp-add-del-split (L.map Thms-single*
l1)
 (L.map Thms-single l2)
 (L.map Thms-single l3)

definition *meth-gen-simp-add-split* l1 l2 = *Method-simp-add-del-split* (L.map *Thms-single* l1)
[]
(L.map *Thms-single* l2)
definition *meth-gen-simp-only* l = *Method-simp-only* (L.map *Thms-single* l)
definition *meth-gen-simp-only'* l = *Method-simp-only* (L.map *Thms-mult* l)
definition *meth-gen-simp-add0* l = *Method-simp-add-del* (L.map *Thms-single* l) []
definition *simp* = *Method-one meth-gen-simp*
definition *simp-add2* l1 l2 = *Method-one (meth-gen-simp-add2 l1 l2)*
definition *simp-add-del* l1 l2 = *Method-one (meth-gen-simp-add-del l1 l2)*
definition *simp-add-del-split* l1 l2 l3 = *Method-one (meth-gen-simp-add-del-split l1 l2 l3)*
definition *simp-add-split* l1 l2 = *Method-one (meth-gen-simp-add-split l1 l2)*
definition *simp-only* l = *Method-one (meth-gen-simp-only l)*
definition *simp-only'* l = *Method-one (meth-gen-simp-only' l)*
definition *simp-add0* l = *Method-one (meth-gen-simp-add0 l)*
definition *simp-add* = *simp-add2* []
definition *simp-all* = *Method-all meth-gen-simp*
definition *simp-all-add* l = *Method-all (meth-gen-simp-add2 [] l)*
definition *simp-all-only* l = *Method-all (meth-gen-simp-only l)*
definition *simp-all-only'* l = *Method-all (meth-gen-simp-only' l)*
definition *auto-simp-add2* l1 l2 = *Method-auto-simp-add-split* (L.flatten [L.map *Thms-mult* l1
, L.map (*Thms-single o Thm-thm*) l2]) []
definition *auto-simp-add-split* l = *Method-auto-simp-add-split* (L.map *Thms-single* l)
definition *rename-tac* = *Method-rename-tac*
definition *case-tac* = *Method-case-tac*
definition *blast* = *Method-blast*
definition *clarify* = *Method-clarify*
definition *metis* = *Method-metis* []
definition *metis0* = *Method-metis*

definition *subst-asm* b = *subst-l0* b [⟨0⟩]
definition *subst* = *subst-l* [⟨0⟩]
definition *auto-simp-add* = *auto-simp-add2* []
definition *auto* = *auto-simp-add* []
end

lemmas [code] =
— def
M.Method-simp-add-del-def
M.Method-subst-l-def
M.rule'-def
M.rule-def
M.drule-def
M.erule-def
M.intro-def
M.elim-def
M.subst-l0-def
M.subst-l-def
M.insert-def
M.plus-def

M.option-def
M.or-def
M.meth-gen-simp-def
M.meth-gen-simp-add2-def
M.meth-gen-simp-add-del-def
M.meth-gen-simp-add-del-split-def
M.meth-gen-simp-add-split-def
M.meth-gen-simp-only-def
M.meth-gen-simp-only'-def
M.meth-gen-simp-add0-def
M.simp-def
M.simp-add2-def
M.simp-add-del-def
M.simp-add-del-split-def
M.simp-add-split-def
M.simp-only-def
M.simp-only'-def
M.simp-add0-def
M.simp-add-def
M.simp-all-def
M.simp-all-add-def
M.simp-all-only-def
M.simp-all-only'-def
M.auto-simp-add2-def
M.auto-simp-add-split-def
M.rename-tac-def
M.case-tac-def
M.blast-def
M.clarify-def
M.metis-def
M.metis0-def
M.subst-asm-def
M.subst-def
M.auto-simp-add-def
M.auto-def

definition *ty-arrow* $l = (\text{case rev } l \text{ of } x \# xs \Rightarrow \text{List.fold Ty-arrow } xs \ x)$

locale *C*

begin

definition *done* = *Command-done*

definition *by* = *Command-by*

definition *sorry* = *Command-sorry*

definition *apply-end* = *Command-apply-end*

definition *apply* = *Command-apply*

definition *using* = *Command-using* o *L.map Thms-single*

definition *unfolding* = *Command-unfolding* o *L.map Thms-single*

definition *let'* = *Command-let*

definition *fix-let* = *Command-fix-let*

definition *fix l = Command-fix-let l [] None []*
definition *have n = Command-have n False*
definition *have0 = Command-have*
end

lemmas [*code*] =
— def
C.done-def
C.by-def
C.sorry-def
C.apply-end-def
C.apply-def
C.using-def
C.unfolding-def
C.let'-def
C.fix-let-def
C.fix-def
C.have-def
C.have0-def

fun *cross-abs-aux* **where**
cross-abs-aux f l x = (λ (Suc n, Abs s - t) ⇒ f s (cross-abs-aux f (s # l) (n, t))
| *(-, e) ⇒ Term-term l e)*
x

definition *cross-abs f n l = cross-abs-aux f [] (n, l)*

2.3.3. Operations of Fold, Map, ..., on the Meta-Model

definition *map-lemma f = (λ Theory-lemma x ⇒ Theory-lemma (f x)*
| *x ⇒ x)*

end

3. Parsing Meta-Models

3.1. Initializing the Parser

```
theory Parser-init
imports ../Init
begin
```

3.1.1. Some Generic Combinators

```
definition K x - = x
```

```
definition co1 = (o)
```

```
definition co2 f g x1 x2 = f (g x1 x2)
```

```
definition co3 f g x1 x2 x3 = f (g x1 x2 x3)
```

```
definition co4 f g x1 x2 x3 x4 = f (g x1 x2 x3 x4)
```

```
definition co5 f g x1 x2 x3 x4 x5 = f (g x1 x2 x3 x4 x5)
```

```
definition co6 f g x1 x2 x3 x4 x5 x6 = f (g x1 x2 x3 x4 x5 x6)
```

```
definition co7 f g x1 x2 x3 x4 x5 x6 x7 = f (g x1 x2 x3 x4 x5 x6 x7)
```

```
definition co8 f g x1 x2 x3 x4 x5 x6 x7 x8 = f (g x1 x2 x3 x4 x5 x6 x7 x8)
```

```
definition co9 f g x1 x2 x3 x4 x5 x6 x7 x8 x9 = f (g x1 x2 x3 x4 x5 x6 x7 x8 x9)
```

```
definition co10 f g x1 x2 x3 x4 x5 x6 x7 x8 x9 x10 = f (g x1 x2 x3 x4 x5 x6 x7 x8 x9 x10)
```

```
definition co11 f g x1 x2 x3 x4 x5 x6 x7 x8 x9 x10 x11 = f (g x1 x2 x3 x4 x5 x6 x7 x8 x9 x10 x11)
```

```
definition co12 f g x1 x2 x3 x4 x5 x6 x7 x8 x9 x10 x11 x12 = f (g x1 x2 x3 x4 x5 x6 x7 x8 x9 x10 x11 x12)
```

```
definition co13 f g x1 x2 x3 x4 x5 x6 x7 x8 x9 x10 x11 x12 x13 = f (g x1 x2 x3 x4 x5 x6 x7 x8 x9 x10 x11 x12 x13)
```

```
definition co14 f g x1 x2 x3 x4 x5 x6 x7 x8 x9 x10 x11 x12 x13 x14 = f (g x1 x2 x3 x4 x5 x6 x7 x8 x9 x10 x11 x12 x13 x14)
```

```
definition co15 f g x1 x2 x3 x4 x5 x6 x7 x8 x9 x10 x11 x12 x13 x14 x15 = f (g x1 x2 x3 x4 x5 x6 x7 x8 x9 x10 x11 x12 x13 x14 x15)
```

```
definition ap1 a v0 f1 v1 = a v0 [f1 v1]
```

```
definition ap2 a v0 f1 f2 v1 v2 = a v0 [f1 v1, f2 v2]
```

```
definition ap3 a v0 f1 f2 f3 v1 v2 v3 = a v0 [f1 v1, f2 v2, f3 v3]
```

```
definition ap4 a v0 f1 f2 f3 f4 v1 v2 v3 v4 = a v0 [f1 v1, f2 v2, f3 v3, f4 v4]
```

```
definition ap5 a v0 f1 f2 f3 f4 f5 v1 v2 v3 v4 v5 = a v0 [f1 v1, f2 v2, f3 v3, f4 v4, f5 v5]
```

```
definition ap6 a v0 f1 f2 f3 f4 f5 f6 v1 v2 v3 v4 v5 v6 = a v0 [f1 v1, f2 v2, f3 v3, f4 v4, f5 v5, f6 v6]
```

```
definition ap7 a v0 f1 f2 f3 f4 f5 f6 f7 v1 v2 v3 v4 v5 v6 v7 = a v0 [f1 v1, f2 v2, f3 v3, f4 v4, f5 v5, f6 v6, f7 v7]
```

```
definition ap8 a v0 f1 f2 f3 f4 f5 f6 f7 f8 v1 v2 v3 v4 v5 v6 v7 v8 = a v0 [f1 v1, f2 v2, f3 v3,
```

$f_4 v_4, f_5 v_5, f_6 v_6, f_7 v_7, f_8 v_8]$

definition $ap9$ $a v_0 f_1 f_2 f_3 f_4 f_5 f_6 f_7 f_8 f_9 v_1 v_2 v_3 v_4 v_5 v_6 v_7 v_8 v_9 = a v_0 [f_1 v_1, f_2 v_2, f_3 v_3, f_4 v_4, f_5 v_5, f_6 v_6, f_7 v_7, f_8 v_8, f_9 v_9]$

definition $ap10$ $a v_0 f_1 f_2 f_3 f_4 f_5 f_6 f_7 f_8 f_9 f_{10} v_1 v_2 v_3 v_4 v_5 v_6 v_7 v_8 v_9 v_{10} = a v_0 [f_1 v_1, f_2 v_2, f_3 v_3, f_4 v_4, f_5 v_5, f_6 v_6, f_7 v_7, f_8 v_8, f_9 v_9, f_{10} v_{10}]$

definition $ap11$ $a v_0 f_1 f_2 f_3 f_4 f_5 f_6 f_7 f_8 f_9 f_{10} f_{11} v_1 v_2 v_3 v_4 v_5 v_6 v_7 v_8 v_9 v_{10} v_{11} = a v_0 [f_1 v_1, f_2 v_2, f_3 v_3, f_4 v_4, f_5 v_5, f_6 v_6, f_7 v_7, f_8 v_8, f_9 v_9, f_{10} v_{10}, f_{11} v_{11}]$

definition $ap12$ $a v_0 f_1 f_2 f_3 f_4 f_5 f_6 f_7 f_8 f_9 f_{10} f_{11} f_{12} v_1 v_2 v_3 v_4 v_5 v_6 v_7 v_8 v_9 v_{10} v_{11} v_{12} = a v_0 [f_1 v_1, f_2 v_2, f_3 v_3, f_4 v_4, f_5 v_5, f_6 v_6, f_7 v_7, f_8 v_8, f_9 v_9, f_{10} v_{10}, f_{11} v_{11}, f_{12} v_{12}]$

definition $ap13$ $a v_0 f_1 f_2 f_3 f_4 f_5 f_6 f_7 f_8 f_9 f_{10} f_{11} f_{12} f_{13} v_1 v_2 v_3 v_4 v_5 v_6 v_7 v_8 v_9 v_{10} v_{11} v_{12} v_{13} = a v_0 [f_1 v_1, f_2 v_2, f_3 v_3, f_4 v_4, f_5 v_5, f_6 v_6, f_7 v_7, f_8 v_8, f_9 v_9, f_{10} v_{10}, f_{11} v_{11}, f_{12} v_{12}, f_{13} v_{13}]$

definition $ap14$ $a v_0 f_1 f_2 f_3 f_4 f_5 f_6 f_7 f_8 f_9 f_{10} f_{11} f_{12} f_{13} f_{14} v_1 v_2 v_3 v_4 v_5 v_6 v_7 v_8 v_9 v_{10} v_{11} v_{12} v_{13} v_{14} = a v_0 [f_1 v_1, f_2 v_2, f_3 v_3, f_4 v_4, f_5 v_5, f_6 v_6, f_7 v_7, f_8 v_8, f_9 v_9, f_{10} v_{10}, f_{11} v_{11}, f_{12} v_{12}, f_{13} v_{13}, f_{14} v_{14}]$

definition $ap15$ $a v_0 f_1 f_2 f_3 f_4 f_5 f_6 f_7 f_8 f_9 f_{10} f_{11} f_{12} f_{13} f_{14} f_{15} v_1 v_2 v_3 v_4 v_5 v_6 v_7 v_8 v_9 v_{10} v_{11} v_{12} v_{13} v_{14} v_{15} = a v_0 [f_1 v_1, f_2 v_2, f_3 v_3, f_4 v_4, f_5 v_5, f_6 v_6, f_7 v_7, f_8 v_8, f_9 v_9, f_{10} v_{10}, f_{11} v_{11}, f_{12} v_{12}, f_{13} v_{13}, f_{14} v_{14}, f_{15} v_{15}]$

definition $ar1$ $a v_0 z = a v_0 [z]$

definition $ar2$ $a v_0 f_1 v_1 z = a v_0 [f_1 v_1, z]$

definition $ar3$ $a v_0 f_1 f_2 v_1 v_2 z = a v_0 [f_1 v_1, f_2 v_2, z]$

definition $ar4$ $a v_0 f_1 f_2 f_3 v_1 v_2 v_3 z = a v_0 [f_1 v_1, f_2 v_2, f_3 v_3, z]$

definition $ar5$ $a v_0 f_1 f_2 f_3 f_4 v_1 v_2 v_3 v_4 z = a v_0 [f_1 v_1, f_2 v_2, f_3 v_3, f_4 v_4, z]$

definition $ar6$ $a v_0 f_1 f_2 f_3 f_4 f_5 v_1 v_2 v_3 v_4 v_5 z = a v_0 [f_1 v_1, f_2 v_2, f_3 v_3, f_4 v_4, f_5 v_5, z]$

definition $ar7$ $a v_0 f_1 f_2 f_3 f_4 f_5 f_6 v_1 v_2 v_3 v_4 v_5 v_6 z = a v_0 [f_1 v_1, f_2 v_2, f_3 v_3, f_4 v_4, f_5 v_5, f_6 v_6, z]$

definition $ar8$ $a v_0 f_1 f_2 f_3 f_4 f_5 f_6 f_7 v_1 v_2 v_3 v_4 v_5 v_6 v_7 z = a v_0 [f_1 v_1, f_2 v_2, f_3 v_3, f_4 v_4, f_5 v_5, f_6 v_6, f_7 v_7, z]$

definition $ar9$ $a v_0 f_1 f_2 f_3 f_4 f_5 f_6 f_7 f_8 v_1 v_2 v_3 v_4 v_5 v_6 v_7 v_8 z = a v_0 [f_1 v_1, f_2 v_2, f_3 v_3, f_4 v_4, f_5 v_5, f_6 v_6, f_7 v_7, f_8 v_8, z]$

definition $ar10$ $a v_0 f_1 f_2 f_3 f_4 f_5 f_6 f_7 f_8 f_9 v_1 v_2 v_3 v_4 v_5 v_6 v_7 v_8 v_9 z = a v_0 [f_1 v_1, f_2 v_2, f_3 v_3, f_4 v_4, f_5 v_5, f_6 v_6, f_7 v_7, f_8 v_8, f_9 v_9, z]$

definition $ar11$ $a v_0 f_1 f_2 f_3 f_4 f_5 f_6 f_7 f_8 f_9 f_{10} v_1 v_2 v_3 v_4 v_5 v_6 v_7 v_8 v_9 v_{10} z = a v_0 [f_1 v_1, f_2 v_2, f_3 v_3, f_4 v_4, f_5 v_5, f_6 v_6, f_7 v_7, f_8 v_8, f_9 v_9, f_{10} v_{10}, z]$

definition $ar12$ $a v_0 f_1 f_2 f_3 f_4 f_5 f_6 f_7 f_8 f_9 f_{10} f_{11} v_1 v_2 v_3 v_4 v_5 v_6 v_7 v_8 v_9 v_{10} v_{11} z = a v_0 [f_1 v_1, f_2 v_2, f_3 v_3, f_4 v_4, f_5 v_5, f_6 v_6, f_7 v_7, f_8 v_8, f_9 v_9, f_{10} v_{10}, f_{11} v_{11}, z]$

definition $ar13$ $a v_0 f_1 f_2 f_3 f_4 f_5 f_6 f_7 f_8 f_9 f_{10} f_{11} f_{12} v_1 v_2 v_3 v_4 v_5 v_6 v_7 v_8 v_9 v_{10} v_{11} v_{12} z = a v_0 [f_1 v_1, f_2 v_2, f_3 v_3, f_4 v_4, f_5 v_5, f_6 v_6, f_7 v_7, f_8 v_8, f_9 v_9, f_{10} v_{10}, f_{11} v_{11}, f_{12} v_{12}, z]$

definition $ar14$ $a v_0 f_1 f_2 f_3 f_4 f_5 f_6 f_7 f_8 f_9 f_{10} f_{11} f_{12} f_{13} v_1 v_2 v_3 v_4 v_5 v_6 v_7 v_8 v_9 v_{10} v_{11} v_{12} v_{13} z = a v_0 [f_1 v_1, f_2 v_2, f_3 v_3, f_4 v_4, f_5 v_5, f_6 v_6, f_7 v_7, f_8 v_8, f_9 v_9, f_{10} v_{10}, f_{11} v_{11}, f_{12} v_{12}, f_{13} v_{13}, z]$

definition $ar15$ $a v_0 f_1 f_2 f_3 f_4 f_5 f_6 f_7 f_8 f_9 f_{10} f_{11} f_{12} f_{13} f_{14} v_1 v_2 v_3 v_4 v_5 v_6 v_7 v_8 v_9 v_{10} v_{11} v_{12} v_{13} v_{14} z = a v_0 [f_1 v_1, f_2 v_2, f_3 v_3, f_4 v_4, f_5 v_5, f_6 v_6, f_7 v_7, f_8 v_8, f_9 v_9, f_{10} v_{10}, f_{11} v_{11}, f_{12} v_{12}, f_{13} v_{13}, f_{14} v_{14}, z]$

3.1.2. Generic Locale for Parsing

```
locale Parse =
  fixes ext :: string ⇒ string

  — (effective) first order
  fixes of-string :: ('a ⇒ 'a list ⇒ 'a) ⇒ (string ⇒ 'a) ⇒ string ⇒ 'a
  fixes of-stringbase :: ('a ⇒ 'a list ⇒ 'a) ⇒ (string ⇒ 'a) ⇒ stringbase ⇒ 'a
  fixes of-nat :: ('a ⇒ 'a list ⇒ 'a) ⇒ (string ⇒ 'a) ⇒ natural ⇒ 'a
  fixes of-unit :: (string ⇒ 'a) ⇒ unit ⇒ 'a
  fixes of-bool :: (string ⇒ 'a) ⇒ bool ⇒ 'a

  — (simulation) higher order
  fixes Of-Pair Of-Nil Of-Cons Of-None Of-Some :: string
begin

definition of-pair a b f1 f2 = (λf. λ(c, d) ⇒ f c d)
  (ap2 a (b Of-Pair) f1 f2)

definition of-list a b f = (λf0. rec-list f0 o co1 K)
  (b Of-Nil)
  (ar2 a (b Of-Cons) f)

definition of-option a b f = rec-option
  (b Of-None)
  (ap1 a (b Of-Some) f)

end

lemmas [code] =
  Parse.of-pair-def
  Parse.of-list-def
  Parse.of-option-def
```

This theory and all the deriving one could also be prefixed by “print” instead of “parse”. In any case, we are converting (or printing) the above datatypes to another format, and finally this format will be “parsed” by Isabelle!

end

3.2. Instantiating the Parser of (Pure) Term

```
theory Parser-Pure
imports Meta-Pure
  Parser-init
begin
```

3.2.1. Main

```
context Parse
```

begin

definition *of-pure-indexname* $a\ b = \text{of-pair } a\ b\ (\text{of-string } a\ b)\ (\text{of-nat } a\ b)$

definition *of-pure-class* $= \text{of-string}$

definition *of-pure-sort* $a\ b = \text{of-list } a\ b\ (\text{of-pure-class } a\ b)$

definition *of-pure-ty* $a\ b = \text{rec-ty}$

$(\text{ap2 } a\ (b\ \langle \text{Type} \rangle)\ (\text{of-string } a\ b)\ (\text{of-list } a\ b\ \text{snd}))$

$(\text{ap2 } a\ (b\ \langle \text{TFree} \rangle)\ (\text{of-string } a\ b)\ (\text{of-pure-sort } a\ b))$

$(\text{ap2 } a\ (b\ \langle \text{TVar} \rangle)\ (\text{of-pure-indexname } a\ b)\ (\text{of-pure-sort } a\ b))$

definition *of-pure-term* $a\ b = (\lambda f0\ f1\ f2\ f3\ f4\ f5. \text{rec-term } f0\ f1\ f2\ f3\ (\text{co2 } K\ f4)\ (\lambda - . f5))$

$(\text{ap2 } a\ (b\ \langle \text{Const} \rangle)\ (\text{of-string } a\ b)\ (\text{of-pure-ty } a\ b))$

$(\text{ap2 } a\ (b\ \langle \text{Free} \rangle)\ (\text{of-string } a\ b)\ (\text{of-pure-ty } a\ b))$

$(\text{ap2 } a\ (b\ \langle \text{Var} \rangle)\ (\text{of-pure-indexname } a\ b)\ (\text{of-pure-ty } a\ b))$

$(\text{ap1 } a\ (b\ \langle \text{Bound} \rangle)\ (\text{of-nat } a\ b))$

$(\text{ar3 } a\ (b\ \langle \text{Abs} \rangle)\ (\text{of-string } a\ b)\ (\text{of-pure-ty } a\ b))$

$(\text{ar2 } a\ (b\ \langle \text{App} \rangle)\ \text{id})$

end

lemmas *[code]* $=$

Parse.of-pure-indexname-def

Parse.of-pure-class-def

Parse.of-pure-sort-def

Parse.of-pure-ty-def

Parse.of-pure-term-def

end

4. Printing Meta-Models

4.1. Initializing the Printer

```
theory Printer-init
imports ../Init
      ../isabelle-home/src/HOL/Isabelle-Main1
begin
```

At the time of writing, the following target languages supported by Isabelle are also supported by the meta-compiler: Haskell, OCaml, Scala, SML.

4.1.1. Kernel Code for Target Languages

```
lazy-code-printing code-module CodeType  $\rightarrow$  (Haskell)  $\langle$ 
module CodeType where {
  type MInt = Integer
; type MMonad a = IO a
}  $\rangle$  | code-module CodeConst  $\rightarrow$  (Haskell)  $\langle$ 
module CodeConst where {
  import System.Directory
; import System.IO
; import qualified CodeConst.Printf

; outFile1 f file = (do
  fileExists <- doesFileExist file
  if fileExists then error (File exists ++ file ++ \n) else do
    h <- openFile file WriteMode
    f (\pat -> hPutStr h . CodeConst.Printf.sprintf1 pat)
    hClose h)

; outStand1 :: ((String -> String -> IO ()) -> IO ()) -> IO ()
; outStand1 f = f (\pat -> putStr . CodeConst.Printf.sprintf1 pat)
}  $\rangle$  | code-module CodeConst.Monad  $\rightarrow$  (Haskell)  $\langle$ 
module CodeConst.Monad where {
  bind a = (>>=) a
; return :: a -> IO a
; return = Prelude.return
}  $\rangle$  | code-module CodeConst.Printf  $\rightarrow$  (Haskell)  $\langle$ 
module CodeConst.Printf where {
  import Text.Printf
; sprintf0 = id
```

```

; sprintf1 :: PrintfArg a => String -> a -> String
; sprintf1 = printf

; sprintf2 :: PrintfArg a => PrintfArg b => String -> a -> b -> String
; sprintf2 = printf

; sprintf3 :: PrintfArg a => PrintfArg b => PrintfArg c => String -> a -> b -> c ->
String
; sprintf3 = printf

; sprintf4 :: PrintfArg a => PrintfArg b => PrintfArg c => PrintfArg d => String -> a ->
b -> c -> d -> String
; sprintf4 = printf

; sprintf5 :: PrintfArg a => PrintfArg b => PrintfArg c => PrintfArg d => PrintfArg e =>
String -> a -> b -> c -> d -> e -> String
; sprintf5 = printf
} > | code-module CodeConst.String -> (Haskell) <
module CodeConst.String where {
  concat s [] = []
; concat s (x : xs) = x ++ concatMap ((++) s) xs
} > | code-module CodeConst.Sys -> (Haskell) <
module CodeConst.Sys where {
  import System.Directory
; isDirectory2 = doesDirectoryExist
} > | code-module CodeConst.To -> (Haskell) <
module CodeConst.To where {
  nat = id
} > | code-module -> (OCaml) <
module CodeType = struct
  type mlInt = int

  type 'a mlMonad = 'a option
end

module CodeConst = struct
  let outFile1 f file =
    try
      let () = if Sys.file-exists file then Printf.eprintf File exists \"%S\\n file else () in
      let oc = open-out file in
      let b = f (fun s a -> try Some (Printf.fprintf oc s a) with - -> None) in
      let () = close-out oc in
      b
    with - -> None

  let outStand1 f =
    f (fun s a -> try Some (Printf.fprintf stdout s a) with - -> None)

  module Monad = struct

```

```

let bind = function
  None -> fun - -> None
  | Some a -> fun f -> f a
let return a = Some a
end

```

```

module Printf = struct
  include Printf
  let sprintf0 = sprintf
  let sprintf1 = sprintf
  let sprintf2 = sprintf
  let sprintf3 = sprintf
  let sprintf4 = sprintf
  let sprintf5 = sprintf
end

```

```

module String = String

```

```

module Sys = struct open Sys
  let isDirectory2 s = try Some (is-directory s) with - -> None
end

```

```

module To = struct
  let nat big-int x = Big-int.int-of-big-int (big-int x)
end
end

```

```

› | code-module → (Scala) ‹
object CodeType {
  type mlMonad [A] = Option [A]
  type mlInt = Int
}

```

```

object CodeConst {
  def outFile1 [A] (f : (String => A => Option [Unit]) => Option [Unit], file0 : String) :
  Option [Unit] = {
    val file = new java.io.File (file0)
    file .isFile match {
      case true => None
      case false =>
        val writer = new java.io.PrintWriter (file)
        f ((fmt : String) => (s : A) => Some (writer .write (fmt .format (s))))
        Some (writer .close ())
    }
  }
}

```

```

def outStand1 [A] (f : (String => A => Option [Unit]) => Option [Unit]) : Option[Unit] =
{
  f ((fmt : String) => (s : A) => Some (print (fmt .format (s))))
}

```

```

}

object Monad {
  def bind [A, B] (x : Option [A], f : A => Option [B]) : Option [B] = x match {
    case None => None
    case Some (a) => f (a)
  }
  def Return [A] (a : A) = Some (a)
}

object Printf {
  def sprintf0 (x0 : String) = x0
  def sprintf1 [A1] (fmt : String, x1 : A1) = fmt .format (x1)
  def sprintf2 [A1, A2] (fmt : String, x1 : A1, x2 : A2) = fmt .format (x1, x2)
  def sprintf3 [A1, A2, A3] (fmt : String, x1 : A1, x2 : A2, x3 : A3) = fmt .format (x1, x2,
x3)
  def sprintf4 [A1, A2, A3, A4] (fmt : String, x1 : A1, x2 : A2, x3 : A3, x4 : A4) = fmt
.format (x1, x2, x3, x4)
  def sprintf5 [A1, A2, A3, A4, A5] (fmt : String, x1 : A1, x2 : A2, x3 : A3, x4 : A4, x5 :
A5) = fmt .format (x1, x2, x3, x4, x5)
}

object String {
  def concat (s : String, l : List [String]) = l filter (- .nonEmpty) mkString s
}

object Sys {
  def isDirectory2 (s : String) = Some (new java.io.File (s) .isDirectory)
}

object To {
  def nat [A] (f : A => BigInt, x : A) = f (x) .intValue ()
}
}

```

```

› | code-module ↪ (SML) ‹
structure CodeType = struct
  type mlInt = string
  type 'a mlMonad = 'a option
end

```

```

structure CodeConst = struct
  structure Monad = struct
    val bind = fn
      NONE => (fn - => NONE)
    | SOME a => fn f => f a
    val return = SOME
  end
end

```

```

structure Printf = struct
  local
    fun sprintf s l =
      case String.fields (fn #% => true | - => false) s of

```

```

[] =>
| [x] => x
| x :: xs =>
  let fun aux acc l-pat l-s =
      case l-pat of
      [] => rev acc
      | x :: xs => aux (String.extract (x, 1, NONE) :: hd l-s :: acc) xs (tl l-s) in
      String.concat (x :: aux [] xs l)
  end
in
  fun sprintf0 s-pat = s-pat
  fun sprintf1 s-pat s1 = sprintf s-pat [s1]
  fun sprintf2 s-pat s1 s2 = sprintf s-pat [s1, s2]
  fun sprintf3 s-pat s1 s2 s3 = sprintf s-pat [s1, s2, s3]
  fun sprintf4 s-pat s1 s2 s3 s4 = sprintf s-pat [s1, s2, s3, s4]
  fun sprintf5 s-pat s1 s2 s3 s4 s5 = sprintf s-pat [s1, s2, s3, s4, s5]
end
end

structure String = struct
  val concat = String.concatWith
end

structure Sys = struct
  val isDirectory2 = SOME o File.is-dir o Path.explode handle ERROR - => K NONE
end

structure To = struct
  fun nat f = Int.toString o f
end

fun outFile1 f file =
  let
    val pfile = Path.explode file
    val () = if File.exists pfile then error (File.exists ^ file ^ "\n") else ()
    val oc = Unsynchronized.ref []
    val - = f (fn a => fn b => SOME (oc := Printf.sprintf1 a b :: (Unsynchronized.! oc))) in
    try (fn () => File.write-list pfile (rev (Unsynchronized.! oc))) ()
  end

fun outStand1 f = outFile1 f (Unsynchronized.! stdout-file)
end

```

4.1.2. Interface with Types

datatype *ml-int* = *ML-int*

code-printing type-constructor *ml-int* \rightarrow (*Haskell*) *CodeType.MInt* — syntax!

```

| type-constructor ml-int → (OCaml) CodeType.mlInt
| type-constructor ml-int → (Scala) CodeType.mlInt
| type-constructor ml-int → (SML) CodeType.mlInt

```

datatype *'a ml-monad* = *ML-monad 'a*

code-printing type-constructor *ml-monad* → (*Haskell*) *CodeType.MLMonad* - — syntax!

```

| type-constructor ml-monad → (OCaml) - CodeType.mlMonad
| type-constructor ml-monad → (Scala) CodeType.mlMonad [-]
| type-constructor ml-monad → (SML) - CodeType.mlMonad

```

type-synonym *ml-string* = *String.literal*

4.1.3. Interface with Constants

module *CodeConst*

consts *out-file1* :: ((*ml-string* ⇒ *'α1* ⇒ *unit ml-monad*) — *fprintf* ⇒ *unit ml-monad*) ⇒ *ml-string* ⇒ *unit ml-monad*

code-printing constant *out-file1* → (*Haskell*) *CodeConst.outFile1*

```

| constant out-file1 → (OCaml) CodeConst.outFile1
| constant out-file1 → (Scala) CodeConst.outFile1
| constant out-file1 → (SML) CodeConst.outFile1

```

consts *out-stand1* :: ((*ml-string* ⇒ *'α1* ⇒ *unit ml-monad*) — *fprintf* ⇒ *unit ml-monad*) ⇒ *unit ml-monad*

code-printing constant *out-stand1* → (*Haskell*) *CodeConst.outStand1*

```

| constant out-stand1 → (OCaml) CodeConst.outStand1
| constant out-stand1 → (Scala) CodeConst.outStand1
| constant out-stand1 → (SML) CodeConst.outStand1

```

module *Monad*

consts *bind* :: *'a ml-monad* ⇒ (*'a* ⇒ *'b ml-monad*) ⇒ *'b ml-monad*

code-printing constant *bind* → (*Haskell*) *CodeConst.Monad.bind*

```

| constant bind → (OCaml) CodeConst.Monad.bind
| constant bind → (Scala) CodeConst.Monad.bind
| constant bind → (SML) CodeConst.Monad.bind

```

consts *return* :: *'a* ⇒ *'a ml-monad*

code-printing constant *return* → (*Haskell*) *CodeConst.Monad.return*

```

| constant return → (OCaml) CodeConst.Monad.return
| constant return → (Scala) CodeConst.Monad.Return — syntax!
| constant return → (SML) CodeConst.Monad.return

```

module *Printf*

consts *sprintf0* :: *ml-string* ⇒ *ml-string*

code-printing constant *sprintf0* → (*Haskell*) *CodeConst.Printf.sprintf0*

```

| constant sprintf0 → (OCaml) CodeConst.Printf.sprintf0
| constant sprintf0 → (Scala) CodeConst.Printf.sprintf0
| constant sprintf0 → (SML) CodeConst.Printf.sprintf0

```

consts *sprintf1* :: *ml-string* ⇒ 'α1 ⇒ *ml-string*
code-printing constant *sprintf1* → (*Haskell*) *CodeConst.Printf.sprintf1*
| **constant** *sprintf1* → (*OCaml*) *CodeConst.Printf.sprintf1*
| **constant** *sprintf1* → (*Scala*) *CodeConst.Printf.sprintf1*
| **constant** *sprintf1* → (*SML*) *CodeConst.Printf.sprintf1*

consts *sprintf2* :: *ml-string* ⇒ 'α1 ⇒ 'α2 ⇒ *ml-string*
code-printing constant *sprintf2* → (*Haskell*) *CodeConst.Printf.sprintf2*
| **constant** *sprintf2* → (*OCaml*) *CodeConst.Printf.sprintf2*
| **constant** *sprintf2* → (*Scala*) *CodeConst.Printf.sprintf2*
| **constant** *sprintf2* → (*SML*) *CodeConst.Printf.sprintf2*

consts *sprintf3* :: *ml-string* ⇒ 'α1 ⇒ 'α2 ⇒ 'α3 ⇒ *ml-string*
code-printing constant *sprintf3* → (*Haskell*) *CodeConst.Printf.sprintf3*
| **constant** *sprintf3* → (*OCaml*) *CodeConst.Printf.sprintf3*
| **constant** *sprintf3* → (*Scala*) *CodeConst.Printf.sprintf3*
| **constant** *sprintf3* → (*SML*) *CodeConst.Printf.sprintf3*

consts *sprintf4* :: *ml-string* ⇒ 'α1 ⇒ 'α2 ⇒ 'α3 ⇒ 'α4 ⇒ *ml-string*
code-printing constant *sprintf4* → (*Haskell*) *CodeConst.Printf.sprintf4*
| **constant** *sprintf4* → (*OCaml*) *CodeConst.Printf.sprintf4*
| **constant** *sprintf4* → (*Scala*) *CodeConst.Printf.sprintf4*
| **constant** *sprintf4* → (*SML*) *CodeConst.Printf.sprintf4*

consts *sprintf5* :: *ml-string* ⇒ 'α1 ⇒ 'α2 ⇒ 'α3 ⇒ 'α4 ⇒ 'α5 ⇒ *ml-string*
code-printing constant *sprintf5* → (*Haskell*) *CodeConst.Printf.sprintf5*
| **constant** *sprintf5* → (*OCaml*) *CodeConst.Printf.sprintf5*
| **constant** *sprintf5* → (*Scala*) *CodeConst.Printf.sprintf5*
| **constant** *sprintf5* → (*SML*) *CodeConst.Printf.sprintf5*

module *String*

consts *String-concat* :: *ml-string* ⇒ *ml-string list* ⇒ *ml-string*
code-printing constant *String-concat* → (*Haskell*) *CodeConst.String.concat*
| **constant** *String-concat* → (*OCaml*) *CodeConst.String.concat*
| **constant** *String-concat* → (*Scala*) *CodeConst.String.concat*
| **constant** *String-concat* → (*SML*) *CodeConst.String.concat*

module *Sys*

consts *Sys-is-directory2* :: *ml-string* ⇒ *bool ml-monad*
code-printing constant *Sys-is-directory2* → (*Haskell*) *CodeConst.Sys.isDirectory2*
| **constant** *Sys-is-directory2* → (*OCaml*) *CodeConst.Sys.isDirectory2*
| **constant** *Sys-is-directory2* → (*Scala*) *CodeConst.Sys.isDirectory2*
| **constant** *Sys-is-directory2* → (*SML*) *CodeConst.Sys.isDirectory2*

module *To*

consts *ToNat* :: (*nat* ⇒ *integer*) ⇒ *nat* ⇒ *ml-int*
code-printing constant *ToNat* → (*Haskell*) *CodeConst.To.nat*
| **constant** *ToNat* → (*OCaml*) *CodeConst.To.nat*

```

| constant ToNat → (Scala) CodeConst.To.nat
| constant ToNat → (SML) CodeConst.To.nat

```

4.1.4. Some Notations (I): Raw Translations

```

syntax -sprint0 :: - ⇒ ml-string (⟨sprint0 (-)⟩)
translations sprint0 x' ⇒ CONST sprintf0 x

```

```

syntax -sprint1 :: - ⇒ - ⇒ ml-string (⟨sprint1 (-)⟩)
translations sprint1 x' ⇒ CONST sprintf1 x

```

```

syntax -sprint2 :: - ⇒ - ⇒ ml-string (⟨sprint2 (-)⟩)
translations sprint2 x' ⇒ CONST sprintf2 x

```

```

syntax -sprint3 :: - ⇒ - ⇒ ml-string (⟨sprint3 (-)⟩)
translations sprint3 x' ⇒ CONST sprintf3 x

```

```

syntax -sprint4 :: - ⇒ - ⇒ ml-string (⟨sprint4 (-)⟩)
translations sprint4 x' ⇒ CONST sprintf4 x

```

```

syntax -sprint5 :: - ⇒ - ⇒ ml-string (⟨sprint5 (-)⟩)
translations sprint5 x' ⇒ CONST sprintf5 x

```

4.1.5. Some Notations (II): Polymorphic Cartouches

```

syntax -cartouche-string' :: String.literal
translations -cartouche-string ⇒ -cartouche-string'

```

⟨ML⟩

4.1.6. Generic Locale for Printing

```

locale Print =
  fixes To-string :: string ⇒ ml-string
  fixes To-nat :: nat ⇒ ml-int
begin
  declare[[cartouche-type' = fun_printf]]
end

```

As remark, printing functions (like *sprintf5*...) are currently weakly typed in Isabelle, we will continue the typing using the type system of target languages.

end

4.2. Instantiating the Printer for (Pure) Term

```

theory Printer-Pure
imports Meta-Pure
  Printer-init
begin

```

```

context Print
begin

fun of-pure-term where of-pure-term l e = (λ
  Const s - ⇒ To-string s
  | Free s - ⇒ To-string s
  | App t1 t2 ⇒ ⟨(ℳs) (ℳs)⟩ (of-pure-term l t1) (of-pure-term l t2)
  | Abs s - t ⇒
    let s = To-string s in
    ⟨(λ ℳs. ℳs)⟩ s (of-pure-term (s # l) t)
  | Bound n ⇒ ⟨ℳs⟩ (l ! nat-of-natural n)) e

```

end

```

lemmas [code] =
  — def
  — fun
  Print.of-pure-term.simps

```

end

4.3. Instantiating the Printer for SML

```

theory Printer-SML
imports Meta-SML
         Printer-init

```

begin

```

context Print
begin

```

```

definition of-semi--val-fun = (λ Sval ⇒ ⟨val⟩
  | Sfun ⇒ ⟨fun⟩)

```

```

fun of-semi--term' where ⟨of-semi--term' e = (λ
  SML-string s ⇒ ⟨ℳs⟩ (To-string (escape-sml s))
  | SML-rewrite val-fun e1 symb e2 ⇒ ⟨ℳs ℳs ℳs ℳs⟩ (of-semi--val-fun val-fun) (of-semi--term'
e1) (To-string symb) (of-semi--term' e2)
  | SML-basic l ⇒ ⟨ℳs⟩ (String-concat ⟨ ⟩ (L.map To-string l))
  | SML-binop e1 s e2 ⇒ ⟨ℳs ℳs ℳs⟩ (of-semi--term' e1) (of-semi--term' (SML-basic [s]))
(of-semi--term' e2)
  | SML-annot e s ⇒ ⟨(ℳs:ℳs)⟩ (of-semi--term' e) (To-string s)
  | SML-function l ⇒ ⟨(fn ℳs)⟩ (String-concat ⟨
  | ⟩ (List.map (λ (s1, s2) ⇒ ⟨ℳs => ℳs⟩ (of-semi--term' s1) (of-semi--term' s2)) l))
  | SML-apply e l ⇒ ⟨(ℳs ℳs)⟩ (of-semi--term' e) (String-concat ⟨ ⟩ (List.map (λ e ⇒ ⟨(ℳs)⟩
(of-semi--term' e)) l))
  | SML-paren p-left p-right e ⇒ ⟨ℳsℳsℳs⟩ (To-string p-left) (of-semi--term' e) (To-string
p-right)

```

| *SML-let-open* *s e* ⇒ ⟨*let open* %*s* in %*s* end⟩ (*To-string* *s*) (*of-semi--term'* *e*) *e*⟩

end

lemmas [*code*] =
 — *def*
Print.of-semi--val-fun-def
 — *fun*
Print.of-semi--term'.simps

end

4.4. Instantiating the Printer for Isabelle

theory *Printer-Isabelle*
imports *Meta-Isabelle*
Printer-Pure
Printer-SML

begin

context *Print*
begin

fun *of-semi--typ* **where** *of-semi--typ* *e* = (λ
Typ-base *s* ⇒ *To-string* *s*
 | *Typ-apply* *name l* ⇒ ⟨%*s* %*s*⟩ (*let* *s* = *String-concat* ⟨, ⟩ (*List.map* *of-semi--typ* *l*) in
 case *l* of [-] ⇒ *s* | - ⇒ ⟨(%*s*)⟩ *s*)
 (*of-semi--typ* *name*)
 | *Typ-apply-bin* *s ty1 ty2* ⇒ ⟨%*s* %*s* %*s*⟩ (*of-semi--typ* *ty1*) (*To-string* *s*) (*of-semi--typ* *ty2*)
 | *Typ-apply-paren* *s1 s2 ty* ⇒ ⟨%*s*%*s*%*s*⟩ (*To-string* *s1*) (*of-semi--typ* *ty*) (*To-string* *s2*)) *e*

definition *of-datatype* - = (λ *Datatype* *n l* ⇒
 ⟨*datatype* %*s* = %*s*⟩
 (*To-string* *n*)
 (*String-concat* ⟨
 | ⟩
 (*L.map*
 (λ(*n*,*l*).
 ⟨%*s* %*s*⟩
 (*To-string* *n*)
 (*String-concat* ⟨ ⟩ (*L.map* (λ*x*. ⟨%*s*⟩ (*of-semi--typ* *x*)) *l*))) *l*))

definition *of-type-synonym* - = (λ *Type-synonym* *n v l* ⇒
 ⟨*type-synonym* %*s* = %*s*⟩ (*if* *v* = [] *then*
 To-string *n*
 else
 of-semi--typ (*Typ-apply* (*Typ-base* *n*) (*L.map* *Typ-base* *v*)))
 (*of-semi--typ* *l*))

```

fun of-semi--term where of-semi--term e = (λ
  Term-rewrite e1 symb e2 ⇒ ⟨%s %s %s⟩ (of-semi--term e1) (To-string symb) (of-semi--term
e2)
  | Term-basic l ⇒ ⟨%s⟩ (String-concat ⟨ ⟩ (L.map To-string l))
  | Term-annot e s ⇒ ⟨(%s::%s)⟩ (of-semi--term e) (of-semi--typ s)
  | Term-bind symb e1 e2 ⇒ ⟨(%s%s. %s)⟩ (To-string symb) (of-semi--term e1) (of-semi--term
e2)
  | Term-fun-case e-case l ⇒ ⟨(%s %s)⟩
    (case e-case of None ⇒ ⟨λ⟩
      | Some e ⇒ ⟨case %s of⟩ (of-semi--term e))
    (String-concat ⟨
  | ⟩ (List.map (λ (s1, s2) ⇒ ⟨%s ⇒ %s⟩ (of-semi--term s1) (of-semi--term s2)) l))
  | Term-apply e l ⇒ ⟨%s %s⟩ (of-semi--term e) (String-concat ⟨ ⟩ (List.map (λ e ⇒ ⟨%s⟩
(of-semi--term e)) l))
  | Term-paren p-left p-right e ⇒ ⟨%s%s%s⟩ (To-string p-left) (of-semi--term e) (To-string
p-right)
  | Term-if-then-else e-if e-then e-else ⇒ ⟨if %s then %s else %s⟩ (of-semi--term e-if) (of-semi--term
e-then) (of-semi--term e-else)
  | Term-term l pure ⇒ of-pure-term (L.map To-string l) pure) e

```

```

definition of-type-notation - = (λ Type-notation n e ⇒
  ⟨type-notation %s (%s)⟩ (To-string n) (To-string e))

```

```

definition of-instantiation - = (λ Instantiation n n-def expr ⇒
  let name = To-string n in
  ⟨instantiation %s :: object
begin
  definition %s-%s-def : %s
  instance ..
end⟩
  name
  (To-string n-def)
  name
  (of-semi--term expr))

```

```

definition of-overloading - = (λ Overloading n-c e-c n e ⇒
  ⟨overloading %s ≡ %s
begin
  definition %s : %s
end⟩ (To-string n-c) (of-semi--term e-c) (To-string n) (of-semi--term e))

```

```

definition of-consts - = (λ Consts n ty symb ⇒
  ⟨consts %s :: %s (%s %s)⟩ (To-string n) (of-semi--typ ty) (To-string Consts-value) (To-string
symb))

```

```

definition of-definition - = (λ
  Definition e ⇒ ⟨definition %s⟩ (of-semi--term e)
  | Definition-where1 name (abbrev, prio) e ⇒ ⟨definition %s ((1%s) %d)
where %s⟩ (To-string name) (of-semi--term abbrev) (To-nat prio) (of-semi--term e)

```

| *Definition-where2* name abbrev $e \Rightarrow \langle \text{definition } \%s (\%s) \rangle$
 where $\%s \rangle$ (*To-string name*) (*of-semi--term abbrev*) (*of-semi--term e*)

definition (*of-semi--thm-attribute-aux-gen* :: *String.literal* \times *String.literal* \Rightarrow $- \Rightarrow - \Rightarrow -$) m *lacc*
 $s =$
 (let $s\text{-base} = (\lambda s \text{ lacc}. \langle \%s[\%s] \rangle$ (*To-string s*) (*String-concat* \langle, \rangle (*L.map* $(\lambda(s, x). \langle \%s \%s \rangle s x)$
lacc))) in
 $s\text{-base } s$ ($m \# \text{ lacc}$)

definition *of-semi--thm-attribute-aux-gen-where* $l =$
 ($\langle \text{where} \rangle$, *String-concat* \langle and \rangle (*L.map* $(\lambda(\text{var}, \text{expr}). \langle \%s = \%s \rangle$
 (*To-string var*)
 (*of-semi--term expr*)) l)

definition *of-semi--thm-attribute-aux-gen-of* $l =$
 ($\langle \text{of} \rangle$, *String-concat* \langle \rangle (*L.map* $(\lambda \text{expr}. \langle \%s \rangle$ (*of-semi--term expr*)) l)

fun *of-semi--thm-attribute-aux where* *of-semi--thm-attribute-aux* *lacc* $e =$
 (λ *Thm-thm s* \Rightarrow *To-string s*
 | *Thm-thms s* \Rightarrow *To-string s*

| *Thm-THEN* (*Thm-thm s*) $e2 \Rightarrow$ *of-semi--thm-attribute-aux-gen* ($\langle \text{THEN} \rangle$, *of-semi--thm-attribute-aux*
 \square $e2$) *lacc s*
 | *Thm-THEN* (*Thm-thms s*) $e2 \Rightarrow$ *of-semi--thm-attribute-aux-gen* ($\langle \text{THEN} \rangle$, *of-semi--thm-attribute-aux*
 \square $e2$) *lacc s*
 | *Thm-THEN* $e1 e2 \Rightarrow$ *of-semi--thm-attribute-aux* (($\langle \text{THEN} \rangle$, *of-semi--thm-attribute-aux* \square
 $e2$) $\#$ *lacc*) $e1$

| *Thm-simplified* (*Thm-thm s*) $e2 \Rightarrow$ *of-semi--thm-attribute-aux-gen* ($\langle \text{simplified} \rangle$, *of-semi--thm-attribute-aux*
 \square $e2$) *lacc s*
 | *Thm-simplified* (*Thm-thms s*) $e2 \Rightarrow$ *of-semi--thm-attribute-aux-gen* ($\langle \text{simplified} \rangle$, *of-semi--thm-attribute-aux*
 \square $e2$) *lacc s*
 | *Thm-simplified* $e1 e2 \Rightarrow$ *of-semi--thm-attribute-aux* (($\langle \text{simplified} \rangle$, *of-semi--thm-attribute-aux*
 \square $e2$) $\#$ *lacc*) $e1$

| *Thm-symmetric* (*Thm-thm s*) \Rightarrow *of-semi--thm-attribute-aux-gen* ($\langle \text{symmetric} \rangle$, $\langle \rangle$) *lacc s*
 | *Thm-symmetric* (*Thm-thms s*) \Rightarrow *of-semi--thm-attribute-aux-gen* ($\langle \text{symmetric} \rangle$, $\langle \rangle$) *lacc s*
 | *Thm-symmetric* $e1 \Rightarrow$ *of-semi--thm-attribute-aux* (($\langle \text{symmetric} \rangle$, $\langle \rangle$) $\#$ *lacc*) $e1$

| *Thm-where* (*Thm-thm s*) $l \Rightarrow$ *of-semi--thm-attribute-aux-gen* (*of-semi--thm-attribute-aux-gen-where*
 l) *lacc s*
 | *Thm-where* (*Thm-thms s*) $l \Rightarrow$ *of-semi--thm-attribute-aux-gen* (*of-semi--thm-attribute-aux-gen-where*
 l) *lacc s*
 | *Thm-where* $e1 l \Rightarrow$ *of-semi--thm-attribute-aux* (*of-semi--thm-attribute-aux-gen-where* $l \#$
lacc) $e1$

| *Thm-of* (*Thm-thm s*) $l \Rightarrow$ *of-semi--thm-attribute-aux-gen* (*of-semi--thm-attribute-aux-gen-of*

l) *lacc s*
| *Thm-of (Thm-thms s) l* \Rightarrow *of-semi--thm-attribute-aux-gen (of-semi--thm-attribute-aux-gen-of*
 l) *lacc s*
| *Thm-of e1 l* \Rightarrow *of-semi--thm-attribute-aux (of-semi--thm-attribute-aux-gen-of l # lacc) e1*

| *Thm-OF (Thm-thm s) e2* \Rightarrow *of-semi--thm-attribute-aux-gen (<OF>, of-semi--thm-attribute-aux*
 \square *e2) lacc s*
| *Thm-OF (Thm-thms s) e2* \Rightarrow *of-semi--thm-attribute-aux-gen (<OF>, of-semi--thm-attribute-aux*
 \square *e2) lacc s*
| *Thm-OF e1 e2* \Rightarrow *of-semi--thm-attribute-aux ((<OF>, of-semi--thm-attribute-aux \square e2) #*
lacc) e1) e

definition *of-semi--thm-attribute* = *of-semi--thm-attribute-aux* \square

definition *of-semi--thm* = (λ *Thms-single thy* \Rightarrow *of-semi--thm-attribute thy*
| *Thms-mult thy* \Rightarrow *of-semi--thm-attribute thy*)

definition *of-semi--thm-attribute-l l* = *String-concat* \langle
 \rangle (*L.map of-semi--thm-attribute l*)

definition *of-semi--thm-attribute-l1 l* = *String-concat* \langle \rangle (*L.map of-semi--thm-attribute l*)

definition *of-semi--thm-l l* = *String-concat* \langle \rangle (*L.map of-semi--thm l*)

definition *of-lemmas -* = (λ *Lemmas-simp-thm simp s l* \Rightarrow
 \langle *lemmas%_s%_s = %_s*
(if String.is-empty s then \langle \rangle *else* \langle *%_s* *(To-string s)*
(if simp then \langle *[simp,code-unfold]* \rangle *else* \langle \rangle
(of-semi--thm-attribute-l l)
| *Lemmas-simp-thms s l* \Rightarrow
 \langle *lemmas%_s [simp,code-unfold] = %_s*
(if String.is-empty s then \langle \rangle *else* \langle *%_s* *(To-string s)*
(String-concat \langle
 \rangle (*L.map To-string l*)))

definition (*of-semi--attrib-genA* :: (*semi--thm list* \Rightarrow *String.literal*)
 \Rightarrow *String.literal* \Rightarrow *semi--thm list* \Rightarrow *String.literal*) *f attr l* = — error reflection: to be merged
(if l = [] then
 \langle \rangle
else
 \langle *%_s: %_s* *attr (f l)*)

definition (*of-semi--attrib-genB* :: (*string list* \Rightarrow *String.literal*)
 \Rightarrow *String.literal* \Rightarrow *string list* \Rightarrow *String.literal*) *f attr l* = — error reflection: to be merged
(if l = [] then
 \langle \rangle
else
 \langle *%_s: %_s* *attr (f l)*)

definition *of-semi--attrib* = *of-semi--attrib-genA of-semi--thm-l*

definition *of-semi-attrib1* = *of-semi-attrib-genB* (λl . *String-concat* $\langle \rangle$ (*L.map To-string* *l*))

definition *of-semi-method-simp* (*s* :: — polymorphism weakening needed by **code-reflect** *String.literal*) =

```
( $\lambda$  Method-simp-only l  $\Rightarrow$   $\langle$  %s only: %s  $\rangle$  s (of-semi-thm-l l)
| Method-simp-add-del-split l1 l2 []  $\Rightarrow$   $\langle$  %s %s %s  $\rangle$ 
  s
  (of-semi-attrib  $\langle$  add  $\rangle$  l1)
  (of-semi-attrib  $\langle$  del  $\rangle$  l2)
| Method-simp-add-del-split l1 l2 l3  $\Rightarrow$   $\langle$  %s %s %s %s  $\rangle$ 
  s
  (of-semi-attrib  $\langle$  add  $\rangle$  l1)
  (of-semi-attrib  $\langle$  del  $\rangle$  l2)
  (of-semi-attrib  $\langle$  split  $\rangle$  l3))
```

fun *of-semi-method where of-semi-method expr* = (λ

```
Method-rule o-s  $\Rightarrow$   $\langle$  rule %s  $\rangle$  (case o-s of None  $\Rightarrow$   $\langle$ 
  | Some s  $\Rightarrow$   $\langle$  %s  $\rangle$  (of-semi-thm-attribute s))
| Method-drule s  $\Rightarrow$   $\langle$  drule %s  $\rangle$  (of-semi-thm-attribute s)
| Method-erule s  $\Rightarrow$   $\langle$  erule %s  $\rangle$  (of-semi-thm-attribute s)
| Method-intro l  $\Rightarrow$   $\langle$  intro %s  $\rangle$  (of-semi-thm-attribute-l1 l)
| Method-elim s  $\Rightarrow$   $\langle$  elim %s  $\rangle$  (of-semi-thm-attribute s)
| Method-subst asm l s  $\Rightarrow$ 
  let s-asm = if asm then  $\langle$  (asm)  $\rangle$  else  $\langle$  in
  if L.map String.meta-of-logic l = [STR "0"] then
     $\langle$  subst %s %s  $\rangle$  s-asm (of-semi-thm-attribute s)
  else
     $\langle$  subst %s (%s) %s  $\rangle$  s-asm (String-concat  $\langle \rangle$  (L.map To-string l)) (of-semi-thm-attribute
s)
| Method-insert l  $\Rightarrow$   $\langle$  insert %s  $\rangle$  (of-semi-thm-l l)
| Method-plus t  $\Rightarrow$   $\langle$  (%s) +  $\rangle$  (String-concat  $\langle \rangle$  (List.map of-semi-method t))
| Method-option t  $\Rightarrow$   $\langle$  (%s) ?  $\rangle$  (String-concat  $\langle \rangle$  (List.map of-semi-method t))
| Method-or t  $\Rightarrow$   $\langle$  (%s)  $\rangle$  (String-concat  $\langle | \rangle$  (List.map of-semi-method t))
| Method-one s  $\Rightarrow$  of-semi-method-simp  $\langle$  simp  $\rangle$  s
| Method-all s  $\Rightarrow$  of-semi-method-simp  $\langle$  simp-all  $\rangle$  s
| Method-auto-simp-add-split l-simp l-split  $\Rightarrow$   $\langle$  auto %s %s  $\rangle$ 
  (of-semi-attrib  $\langle$  simp  $\rangle$  l-simp)
  (of-semi-attrib1  $\langle$  split  $\rangle$  l-split)
| Method-rename-tac l  $\Rightarrow$   $\langle$  rename-tac %s  $\rangle$  (String-concat  $\langle \rangle$  (L.map To-string l))
| Method-case-tac e  $\Rightarrow$   $\langle$  case-tac %s  $\rangle$  (of-semi-term e)
| Method-blast None  $\Rightarrow$   $\langle$  blast  $\rangle$ 
| Method-blast (Some n)  $\Rightarrow$   $\langle$  blast %d  $\rangle$  (To-nat n)
| Method-clarify  $\Rightarrow$   $\langle$  clarify  $\rangle$ 
| Method-metis l-opt l  $\Rightarrow$   $\langle$  metis %s %s  $\rangle$  (if l-opt = [] then  $\langle$ 
  else
     $\langle$  (%s)  $\rangle$  (String-concat  $\langle \rangle$  (L.map To-string l-opt)))
(of-semi-thm-attribute-l1 l)) expr
```

definition *of-semi-command-final* = (λ *Command-done* \Rightarrow \langle done \rangle)

| *Command-by l-apply* \Rightarrow $\langle \text{by}(\%s) \rangle$ (*String-concat* \langle , \rangle (*L.map*
of-semi--method l-apply))
| *Command-sorry* \Rightarrow $\langle \text{sorry} \rangle$

definition *of-semi--command-state* = (
 λ *Command-apply-end* $\square \Rightarrow \langle \rangle$
| *Command-apply-end l-apply* $\Rightarrow \langle \text{apply-end}(\%s) \rangle$
 \rangle (*String-concat* \langle , \rangle (*L.map of-semi--method l-apply*)))

definition *of-semi--command-proof* = (
let thesis = $\langle ?thesis \rangle$
; *scope-thesis-gen* = $\lambda \text{proof show when.} \langle \text{proof} - \%s \text{ show } \%s\%s \rangle$
 \rangle *proof*
show
(*if when* = \square *then*
 $\langle \rangle$
else
 $\langle \text{when } \%s \rangle$ (*String-concat* $\langle \rangle$ (*L.map* ($\lambda t. \langle \%s \rangle$ (*of-semi--term t*)) *when*)))
; *scope-thesis* = $\lambda s. \text{scope-thesis-gen } s \text{ thesis } \square$ *in*
 λ *Command-apply* $\square \Rightarrow \langle \rangle$
| *Command-apply l-apply* $\Rightarrow \langle \text{apply}(\%s) \rangle$
 \rangle (*String-concat* \langle , \rangle (*L.map of-semi--method l-apply*))
| *Command-using l* $\Rightarrow \langle \text{using } \%s \rangle$
 \rangle (*of-semi--thm-l l*)
| *Command-unfolding l* $\Rightarrow \langle \text{unfolding } \%s \rangle$
 \rangle (*of-semi--thm-l l*)
| *Command-let e-name e-body* $\Rightarrow \text{scope-thesis} (\langle \text{let } \%s = \%s \rangle$ (*of-semi--term e-name*) (*of-semi--term*
e-body))
| *Command-have n b e e-last* $\Rightarrow \text{scope-thesis} (\langle \text{have } \%s\%s: \%s \%s \rangle$ (*To-string n*) (*if b then*
 $\langle \text{[simp]} \rangle$ *else* $\langle \rangle$) (*of-semi--term e*) (*of-semi--command-final e-last*))
| *Command-fix-let l l-let o-show -* \Rightarrow
scope-thesis-gen
 $\langle \text{fix } \%s\%s \rangle$ (*String-concat* $\langle \rangle$ (*L.map To-string l*))
(*String-concat* \langle
 \rangle (*L.map*
 $\lambda(e\text{-name}, e\text{-body}).$
 \langle *let* $\%s = \%s \rangle$ (*of-semi--term e-name*) (*of-semi--term*
e-body))
l-let))
(*case o-show of None* \Rightarrow *thesis*
| *Some (l-show, -)* $\Rightarrow \langle \%s \rangle$ (*String-concat* $\langle \Longrightarrow \rangle$ (*L.map of-semi--term l-show*)))
(*case o-show of None* $\Rightarrow \square$ | *Some (-, l-when)* \Rightarrow *l-when*))

definition *of-lemma -* =
 λ *Lemma n l-spec l-apply tactic-last* \Rightarrow
 $\langle \text{lemma } \%s : \%s \rangle$
 $\%s\%s \rangle$
(*To-string n*)
(*String-concat* $\langle \Longrightarrow \rangle$ (*L.map of-semi--term l-spec*))

```

    (String-concat <> (L.map (λ [] ⇒ <> | l-apply ⇒ < apply(%s)
    >
    (String-concat <, > (L.map of-semi--method l-apply)))
    l-apply))
    (of-semi--command-final tactic-last)
  | Lemma-assumes n l-spec concl l-apply tactic-last ⇒
    <lemma %s : %s
    %s% %s %s>
    (To-string n)
    (String-concat <> (L.map (λ(n, b, e).
    <
    assumes %s% %s>
    (let (n, b) = if b then (<%s[simp]> (To-string n), False) else (To-string n, String.is-empty
    n) in
    if b then <> else <%s: > n)
    (of-semi--term e)) l-spec
    @@@@
    [ <
    shows %s> (of-semi--term concl)]))
    (String-concat <> (L.map of-semi--command-proof l-apply))
    (of-semi--command-final tactic-last)
    (String-concat < >
    (L.map
    (λl-apply-e.
    <%sqed>
    (if l-apply-e = [] then
    <>
    else
    <
    %s >
    (String-concat <> (L.map of-semi--command-state l-apply-e))))
    (List.map-filter
    (λ Command-let - - ⇒ Some [] | Command-have - - - - ⇒ Some [] | Command-fix-let
    - - - l ⇒ Some l | - ⇒ None)
    (rev l-apply))))))

```

definition *of-axiomatization* - = (λ *Axiomatization* n e ⇒ <axiomatization where %s:
 %s> (To-string n) (of-semi--term e))

definition *of-section* - = (λ *Section* n section-title ⇒
 <%s <%s>>
 (<%ssection> (if n = 0 then <>
 else if n = 1 then <sub>
 else <subsub>))
 (To-string section-title))

definition *of-text* - = (λ *Text* s ⇒ <text <%s>> (To-string s))

definition *of-ML* - = (λ *SML* *e* ⇒ ⟨*ML* ‹%s›⟩ (of-semi--term' *e*))

definition *of-setup* - = (λ *Setup* *e* ⇒ ⟨*setup* ‹%s›⟩ (of-semi--term' *e*))

definition *of-thm* - = (λ *Thm* *thm* ⇒ ⟨*thm* ‹%s›⟩ (of-semi--thm-attribute-l1 *thm*))

definition ‹*of-interpretation* - = (λ *Interpretation* *n* *loc-n* *loc-param* *tac* ⇒
 ‹*interpretation* ‹%s: %s%*s*›
 ‹%s› (To-string *n*)
 (To-string *loc-n*)
 (String-concat ‹› (L.map (λ*s*. ‹%s›) (of-semi--term *s*)) *loc-param*)
 (of-semi--command-final *tac*)›

definition *of-semi--theory env* =
 (λ *Theory-datatype* *dataty* ⇒ *of-datatype env dataty*
 | *Theory-type-synonym* *ty-synonym* ⇒ *of-type-synonym env ty-synonym*
 | *Theory-type-notation* *ty-notation* ⇒ *of-type-notation env ty-notation*
 | *Theory-instantiation* *instantiation-class* ⇒ *of-instantiation env instantiation-class*
 | *Theory-overloading* *overloading* ⇒ *of-overloading env overloading*
 | *Theory-consts* *consts-class* ⇒ *of-consts env consts-class*
 | *Theory-definition* *definition-hol* ⇒ *of-definition env definition-hol*
 | *Theory-lemmas* *lemmas-simp* ⇒ *of-lemmas env lemmas-simp*
 | *Theory-lemma* *lemma-by* ⇒ *of-lemma env lemma-by*
 | *Theory-axiomatization* *axiom* ⇒ *of-axiomatization env axiom*
 | *Theory-section* *section-title* ⇒ *of-section env section-title*
 | *Theory-text* *text* ⇒ *of-text env text*
 | *Theory-ML* *ml* ⇒ *of-ML env ml*
 | *Theory-setup* *setup* ⇒ *of-setup env setup*
 | *Theory-thm* *thm* ⇒ *of-thm env thm*
 | *Theory-interpretation* *thm* ⇒ *of-interpretation env thm*)

definition *String-concat-map* *s f l* = *String-concat s* (L.map *f l*)

definition ‹*of-semi--theories env* =
 (λ *Theories-one* *t* ⇒ *of-semi--theory env t*
 | *Theories-locale* *data* *l* ⇒
 ‹*locale* ‹%s =
 %s
 begin
 %s
 end› (To-string (HolThyLocale-name *data*))
 (String-concat-map
 ‹
 ›
 (λ (*l-fix*, *o-assum*).
 ‹%s%*s*› (String-concat-map ‹
 › (λ(*e*, *ty*). ‹*fixes* ‹%s :: %s› (of-semi--term *e*) (of-semi--typ *ty*)) *l-fix*)
 (case *o-assum* of None ⇒ ‹›
 | Some (*name*, *e*) ⇒ ‹

```

assumes %s: %s› (To-string name) (of-semi--term e)))
      (HolThyLocale-header data))
      (String-concat-map ‹
› (String-concat-map ‹
› (of-semi--theory env)) l))›

```

end

```

lemmas [code] =
  — def
    Print.of-datatype-def
    Print.of-type-synonym-def
    Print.of-type-notation-def
    Print.of-instantiation-def
    Print.of-overloading-def
    Print.of-consts-def
    Print.of-definition-def
    Print.of-semi--thm-attribute-aux-gen-def
    Print.of-semi--thm-attribute-aux-gen-where-def
    Print.of-semi--thm-attribute-aux-gen-of-def
    Print.of-semi--thm-attribute-def
    Print.of-semi--thm-def
    Print.of-semi--thm-attribute-l-def
    Print.of-semi--thm-attribute-l1-def
    Print.of-semi--thm-l-def
    Print.of-lemmas-def
    Print.of-semi--attrib-genA-def
    Print.of-semi--attrib-genB-def
    Print.of-semi--attrib-def
    Print.of-semi--attrib1-def
    Print.of-semi--method-simp-def
    Print.of-semi--command-final-def
    Print.of-semi--command-state-def
    Print.of-semi--command-proof-def
    Print.of-lemma-def
    Print.of-axiomatization-def
    Print.of-section-def
    Print.of-text-def
    Print.of-ML-def
    Print.of-setup-def
    Print.of-thm-def
    Print.of-interpretation-def
    Print.of-semi--theory-def
    Print.String-concat-map-def
    Print.of-semi--theories-def

  — fun

```

Print.of-semi--typ.simps
Print.of-semi--term.simps
Print.of-semi--thm-attribute-aux.simps
Print.of-semi--method.simps

end

5. Main

We present two solutions for obtaining an Isabelle file.

5.1. Static Meta Embedding with Exportation

```
theory Generator-static
imports Printer ../../Antiquote-Setup
begin
```

In the “static” solution: the user manually generates the Isabelle file after writing by hand a Toy input to translate. The input is not written with the syntax of the Toy Language, but with raw Isabelle constructors.

5.1.1. Giving an Input to Translate

```
definition Design =
  (let n = λn1 n2. ToyTyObj (ToyTyCore-pre n1) (case n2 of None ⇒ [] | Some n2 ⇒ [[ToyTyCore-pre
n2]]))
  ; mk = λn l. toy-class-raw.make n l [] False in
  [ mk (n ⟨Galaxy⟩ None) [(⟨sound⟩, ToyTy-raw ⟨unit⟩), (⟨moving⟩, ToyTy-raw ⟨bool⟩)]
  , mk (n ⟨Planet⟩ (Some ⟨Galaxy⟩)) [(⟨weight⟩, ToyTy-raw ⟨nat⟩)]
  , mk (n ⟨Person⟩ (Some ⟨Planet⟩)) [(⟨salary⟩, ToyTy-raw ⟨int⟩)] ]
```

Since we are in a Isabelle session, at this time, it becomes possible to inspect with the command **value** the result of the translations applied with *Design*. A suitable environment should nevertheless be provided, one can typically experiment this by copying-pasting the following environment initialized in the above *main*:

```
definition main =
  (let n = λn1. ToyTyObj (ToyTyCore-pre n1) []
  ; ToyMult = λm r. toy-multiplicity.make [m] r [Set] in
  write-file
  (compiler-env-config.extend
  (compiler-env-config-empty True None (oidInit (Oid 0)) Gen-only-design (None, False)
  (| D-output-disable-thy := False
  , D-output-header-thy := Some (⟨Design-generated⟩
  , [⟨../Toy-Library⟩
  , [⟨../embedding/Generator-dynamic-sequential⟩ ]))
  ( L.map (META-class-raw Floor1) Design
  @@@@ [ META-association (toy-association.make
  ToyAssTy-association
```

```

      (ToyAssRel [ (n ⟨Person⟩, ToyMult (Mult-star, None) None)
                  , (n ⟨Person⟩, ToyMult (Mult-nat 0, Some (Mult-nat 1))
                    (Some ⟨boss⟩))]))
    , META-flush-all ToyFlushAll]
    , None)))

```

5.1.2. Statically Executing the Exportation

```

apply_code_printing ()
export_code main
(* in Haskell *)
(* in OCaml module_name M *)
(* in Scala module_name M *)
(* in SML   module_name M *)

```

After the exportation and executing the exported, we obtain an Isabelle `.thy` file containing the generated code associated to the above input.

`end`

5.2. Dynamic Meta Embedding with Reflection

```

theory Generator-dynamic-sequential
imports Printer
  ../isabelle-home/src/HOL/Isabelle-Main2begin

```

In the “dynamic” solution: the exportation is automatically handled inside Isabelle/jEdit. Inputs are provided using the syntax of the Toy Language, and in output we basically have two options:

- The first is to generate an Isabelle file for inspection or debugging. The generated file can interactively be loaded in Isabelle/jEdit, or saved to the hard disk. This mode is called the “deep exportation” mode or shortly the “deep” mode. The aim is to maximally automate the process one is manually performing in `Generator_static.thy`.
- On the other hand, it is also possible to directly execute in Isabelle/jEdit the generated file from the random access memory. This mode corresponds to the “shallow reflection” mode or shortly “shallow” mode.

In both modes, the reflection is necessary since the main part used by both was defined at Isabelle side. As a consequence, experimentations in “deep” and “shallow” are performed without leaving the editing session, in the same as the one the meta-compiler is actually running.

```

apply-code-printing-reflect ⟨
  val stdout-file = Unsynchronized.ref
  ⟩

```

This variable is not used in this theory (only in `Generator_static.thy`), but needed for well typechecking the reflected SML code.

Part II.

A Toy Example

5.3. A Toy Library for Objects in a State

```
theory Toy-Library
imports Main
begin

type-notation option (⟨⟨-⟩⟩)
notation Some (⟨[-]⟩)

fun drop :: 'α option ⇒ 'α (⟨[-]⟩)
where drop-lift[simp]: [v] = v

type-synonym oid = nat

type-synonym 'α val' = unit ⇒ 'α
type-notation val' (⟨(-)⟩)

record (′ℳ)state =
  heap :: oid → 'ℳ
  assocs :: oid → ((oid list) list) list

lemmas [simp,code-unfold] = state.defs

end
```

5.4. Example: A Class Model Converted into a Theory File

5.4.1. Introduction

```
theory
  Design-deep
imports
  ../embedding/Generator-dynamic-sequential
  ../../Antiquote-Setup
begin
```

In this example, we configure our package to generate a `.thy` file, without executing the associated generated code contained in this `.thy` file (c.f. `Design_shallow.thy` for a direct evaluation). This mode is particularly relevant for debugging purposes: while by default no evaluation occurs, the generated files (and their proofs!) can be executed on a step by step basis, depending on how we interact with the output window (by selectively clicking on what is generated).

After clicking on the generated content, the newly inserted content could depend on some theories which are not loaded by this current one. In this case, it is necessary to manually add all the needed dependencies above after the keyword **imports**. One should compare this current theory with `Design_shallow.thy` to see the differences of imported theories, and which ones to manually import (whenever an error happens).

```
generation-syntax [ ]
```

```
generation_syntax
[ deep
  (generation_semantics [ design ])
  (THEORY Design_generated)
  (IMPORTS ["../Toy_Library", "../Toy_Library_Static"]
           "../embedding/Generator_dynamic_sequential")
  SECTION
  (*SORRY*) (*no_dirty*)
  [ (* in Haskell *)
    (* in OCaml module_name M *)
    (* in Scala module_name M *)
    in SML module_name M ]
  (output_directory "../document_generated")
  (*, syntax_print*) ]
```

While in theory it is possible to set the **deep** mode for generating in all target languages, i.e. by writing [*in Haskell, in OCaml module-name M, in Scala module-name M, in SML module-name M*], usually using only one target is enough, since the task of all target is to generate the same Isabelle content. However in case one language takes too much time to setup, we recommend to try the generation with another target language, because all optimizations are currently not (yet) seemingly implemented for all target languages, or differently activated.

5.4.2. Designing Class Models (I): Basics

The following example shows the definitions of a set of classes, called the “universe” of classes. Instead of providing a single command for building all the complete universe of classes directly in one block, we are constructing classes one by one. So globally the universe describing all classes is partial, it will only be fully constructed when all classes will be finished to be defined.

This allows to define classes without having to follow a particular order of definitions. Here *Atom* is defined before the one of *Molecule* (*Molecule* will come after):

```
Class Atom < Molecule
  Attributes size : Integer
End
```

The “blue” color of **End** indicates that **End** is not a “green” keyword. **End** and **Class** are in fact similar, they belong to the group of meta-commands (all meta-commands are defined in *Isabelle-Meta-Model.Generator-dynamic-sequential*). At run-time and in **deep** mode, the semantics of all meta-commands are approximately similar: all meta-commands displays some quantity of Isabelle code in the output window (as long as

meta-commands are syntactically correctly formed). However each meta-command is unique because what is displayed in the output window depends on the sequence of all meta-commands already encountered before (and also depends on arguments given to the meta-commands).

One particularity of **End** is to behave as the identity function when **End** is called without arguments. As example, here we are calling lots of **End** without arguments, and no Isabelle code is generated.

End End End

We remark that, like any meta-commands, **End** could have been written anywhere in this theory, for example before **Class** or even before **generation-syntax...** Something does not have to be specially opened before using an **End**.

Class *Molecule* < *Person*

As example, here no **End** is written.

The semantics of **End** is further precised here. We earlier mentioned that the universe of classes is partially constructed, but one can still examine what is partially constructed, and one possibility is to use **End** for doing so.

End can be seen as a lazy meta-command:

- without parameters, no code is generated,
- with some parameters (e.g., the symbol !), it forces the generation of the computation of the universe, by considering all already encountered classes. Then a partial representation of the universe can be interactively inspected.

Class *Galaxy*

Attributes *wormhole* : *UnlimitedNatural*
is-sound : *Void*

End!

At this position, in the output window, we can observe for the first time some generated Isabelle code, corresponding to the partial universe of classes being constructed.

Note: By default, *Atom* and *Molecule* are not (yet) present in the shown universe because *Person* has not been defined in a separate line (unlike *Galaxy* above).

Class *Person* < *Galaxy*

Attributes *salary* : *Integer*
boss : *Person*
is-meta-thinking: *Boolean*

There is not only **End** which forces the computation of the universe, for example **Instance** declares a set of objects belonging to the classes earlier defined, but the entire universe is needed as knowledge, so there is no choice than forcing the generation of the universe.

Instance $X_{Person1} :: Person = [salary = 1300 , boss = X_{Person2}]$
and $X_{Person2} :: Person = [salary = 1800]$

Here we will call **Instance** again to show that the universe will not be computed again since it was already computed in the previous **Instance**.

```
Instance  $X_{Person3} :: Person = [ salary = 1 ]$ 
```

However at any time, the universe can (or will) automatically be recomputed, whenever we are adding meanwhile another class:

```
(* Class Big_Bang < Atom (* This will force the creation of a new universe.
*) *)
```

As remark, not only the universe is recomputed, but the recomputation takes also into account all meta-commands already encountered. So in the new setting, $X_{Person1}$, $X_{Person2}$ and $X_{Person3}$ will be resurrected... after the *Big-Bang*.

5.4.3. Designing Class Models (II): Jumping to Another Semantic Floor

Until now, meta-commands was used to generate lines of code, and these lines belong to the Isabelle language. One particularity of meta-commands is to generate pieces of code containing not only Isabelle code but also arbitrary meta-commands. In **deep** mode, this is particularly not a danger for meta-commands to generate themselves (whereas for **shallow** the recursion might not terminate).

In this case, such meta-commands must automatically generate the appropriate call to **generation-syntax** beforehand. However this is not enough, the compiling environment (comprising the history of meta-commands) are changing throughout the interactive evaluations, so the environment must also be taken into account and propagated when meta-commands are generating themselves. For example, the environment is needed for consultation whenever resurrecting objects, recomputing the universe or accessing the hierarchy of classes being defined.

As a consequence, in the next example a line **setup** is added after **generation-syntax** for bootstrapping the state of the compiling environment.

```
State  $\sigma_1 =$ 
  [ ([ salary = 1000 , boss = self 1 ] :: Person)
    , ([ salary = 1200 ] :: Person)

    , ([ salary = 2600 , boss = self 3 ] :: Person)
    ,  $X_{Person1}$ 
    , ([ salary = 2300 , boss = self 2 ] :: Person)

    ,  $X_{Person2}$  ]
```

```
State  $\sigma_1' =$ 
  [  $X_{Person1}$ 
    ,  $X_{Person2}$ 
    ,  $X_{Person3}$  ]
```

In certain circumstances, the command **setup** must be added again between some par-

ticular interleaving of two meta-commands and this may not depend on the presence of **generation-syntax** (which is defined only once when generating the first meta-command). For more details, one can refer to the source code of *ignore-meta-header* and *bootstrap-floor*.

PrePost $\sigma_1 \sigma_1'$

The generation of meta-commands allows to perform various extensions on the Toy language being embedded, without altering the semantics of a particular command. **PrePost** usually only takes “bound variables” as parameters (not arbitrary λ -terms), however the semantics of **PrePost** was extended to mimic the support of some particular terms not restricted to variables. This extension was implemented by executing some steps of “ ζ -reductions rewriting rules” operating on the meta-level of commands. First, it is at least needed to extend the syntax of expressions accepted by **PrePost**, we then modify the parsing so that a larger subset of λ -terms can be given as parameters. Starting from this expression:

```
(* PrePost \<sigma>\<^sub>1 [ ([ salary = 1000 , boss = self 1 ] :: Person)
] *)
```

the rewriting begins with a first call to the next semantic floor, we obtain the following meta-commands (where **PrePost** [*shallow*] is an expression in normal form):

```
(* State WFF_10_post = [ ([ "salary" = 1000, "boss" = self 1 ] :: Person)
]
PrePost[shallow] \<sigma>\<^sub>1 WFF_10_post *) (WFF-10-post is an automat-
```

ically generated name).

The rewriting of the above **State** is performed in its turn. Finally the overall ultimately terminates when reaching **Instance** being already in normal form:

```
(* Instance WFF_10_post_object0 :: Person = [ "salary" = 1000, "boss" = [
] ]
```

```
State[shallow] WFF_10_post = [ WFF_10_post_object0 ]
```

```
PrePost[shallow] \<sigma>\<^sub>1 WFF_10_post *)
```

5.4.4. Designing Class Models (III): Interaction with (Pure) Term

Meta-commands are obviously not restricted to manipulate expressions in the Outer Syntax level. It is possible to build meta-commands so that Inner Syntax expressions are directly parsed. However the dependencies of this theory have been minimized so that experimentations and debugging can easily occur in **deep** mode (this file only depends on *Isabelle-Meta-Model.Generator-dynamic-sequential*). Since the Inner Syntax expressions would perhaps manipulate expressions coming from other theories than *Isabelle-Meta-Model.Generator-dynamic-sequential*, it can be desirable to consider the Inner Syntax container as a string and leave the parsing for subsequent semantic floors. This is what is implemented here:

```
Context Person :: content ()
  Post "><"
```

Here the expression `><` is not well-typed in Isabelle, but an error is not raised because the above expression is not (yet) parsed as an Inner Syntax element¹.

However, this is not the same for the resulting generated meta-command:

```
Context [shallow] Person :: content ()
  Post : "(\\<lambda> result self. (><))"
```

and an error is immediately raised because the parsing of Inner Syntax expressions is activated in this case.

For example, one can put the mouse, with the CTRL gesture, over the variable *a*, *b* or *c* to be convinced that they are free variables compared with above:

```
Context[shallow] Person :: content ()
  Post : a + b = c
```

5.4.5. Designing Class Models (IV): Saving the Generated to File

The experimentations usually finish by saving all the universe and generated Isabelle theory to the hard disk:

```
(* generation_syntax deep flush_all *)
```

5.4.6. Designing Class Models (V): Inspection of Generated Files

According to options given to the (first) command `generation-syntax` above, we retrieve the first generated file in the mentioned directory: `../document_generated/Design_generated.thy`.

Because this file still contains meta-commands, we are here executing again a new generating step inside this file, the new result becomes saved in `../document_generated/Design_generated_generated.thy`. As remark, in this last file, the dependency to *Isabelle-Meta-Model.Generator-dynamic-sequential* was automatically removed because the meta-compiler has detected the absence of meta-commands in the generated content.

Note: While the first generated file is intended to be always well-typed, it can happen that subsequent generations will lead to a not well-typed file. This is because the meta-compiler only saves the history of meta-commands. In case some “native” Isabelle declarations are generated among meta-commands, then these Isabelle declarations are not saved by the meta-compiler, so these declarations will not be again generated. Anyway, we see potential solutions for solving this and they would perhaps be implemented in a future version of the meta-compiler...

end

¹In any case an error will not be raised, because the above code is written in verbatim in the real `.thy` file, however one can copy-paste this code out of the verbatim scope to see that no errors are really raised. For presentation purposes, it was embedded in verbatim because we will later discuss about meta-commands generating Isabelle code, and then what is generated by this meta-command is of course not well-typed!

5.5. Example: A Class Model Interactively Executed

5.5.1. Introduction

```
theory
  Design-shallow
imports
  ../Toy-Library
  ../Toy-Library-Static
  ../embedding/Generator-dynamic-sequential
  ../../Antiquote-Setup
begin
```

In this example, we configure our package to execute tactic SML code (corresponding to some generated `.thy` file, `Design_deep.thy` details how to obtain such generated `.thy` file). Since SML code are already compiled (or reflected) and bound with the native Isabelle API in `Isabelle-Meta-Model.Generator-dynamic-sequential`, nothing is generated in this theory. The system only parses arguments given to meta-commands and immediately calls the corresponding compiled functions.

The execution time is comparatively similar as if tactics were written by hand, except that the generated SML code potentially inherits all optimizations performed by the raw code generation of Isabelle (if any).

```
generation-syntax [ shallow (generation-semantic [ design ])
                  ]
```

The configuration in **shallow** mode is straightforward: in this mode **generation-syntax** basically terminates in $O(1)$.

5.5.2. Designing Class Models (I): Basics

```
Class Atom < Molecule
  Attributes size : Integer
End
```

```
End End End
```

```
Class Molecule < Person
```

```
Class Galaxy
  Attributes wormhole : UnlimitedNatural
             is-sound : Void
End!
```

```
Class Person < Galaxy
  Attributes salary : Integer
             boss : Person
             is-meta-thinking: Boolean
```

```
Instance  $X_{Person1} :: Person = [ salary = 1300 , boss = X_{Person2} ]$   
  and  $X_{Person2} :: Person = [ salary = 1800 ]$ 
```

```
Instance  $X_{Person3} :: Person = [ salary = 1 ]$ 
```

5.5.3. Designing Class Models (II): Jumping to Another Semantic Floor

5.5.4. Designing Class Models (III): Interaction with (Pure) Term

Here in **shallow** mode, the following expression is directly rejected:

```
Context  $Person :: content ()$ 
```

```
  Post "><"
```

```
Context[shallow]  $Person :: content ()$ 
```

```
  Post :  $a + b = c$ 
```

```
end
```

Bibliography

- [1] A. D. Brucker, F. Tuong, and B. Wolff. Featherweight ocl: A proposal for a machine-checked formal semantics for ocl 2.5. *Archive of Formal Proofs*, Jan. 2014. ISSN 2150-914x. URL <http://www.brucker.ch/bibliography/abstract/brucker.ea-featherweight-2014>. http://isa-afp.org/entries/Featherweight_OCL.shtml, Formal proof development.

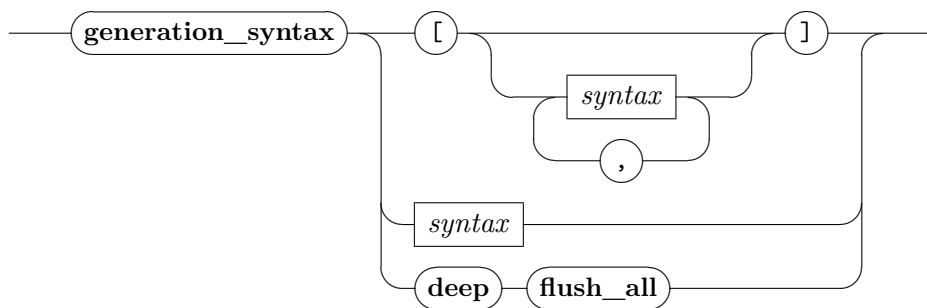
Part III.

Appendix

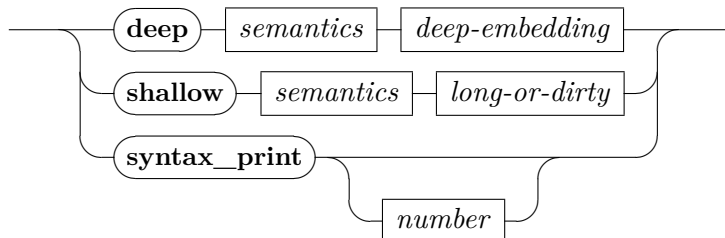
A. Grammars of Commands

A.1. Main Setup of Meta Commands

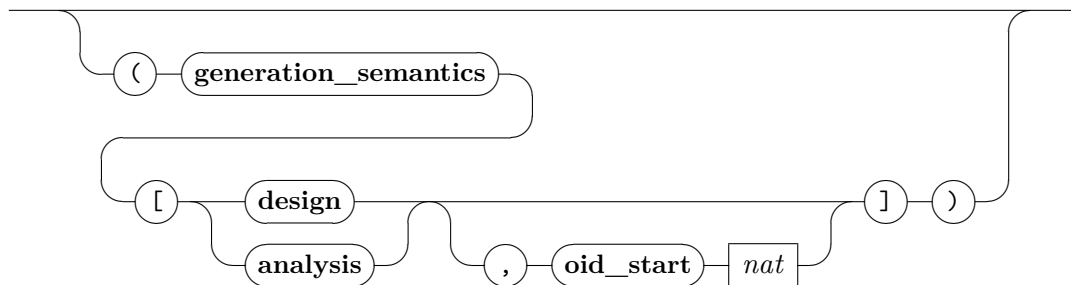
`generation-syntax` : *theory* → *theory*



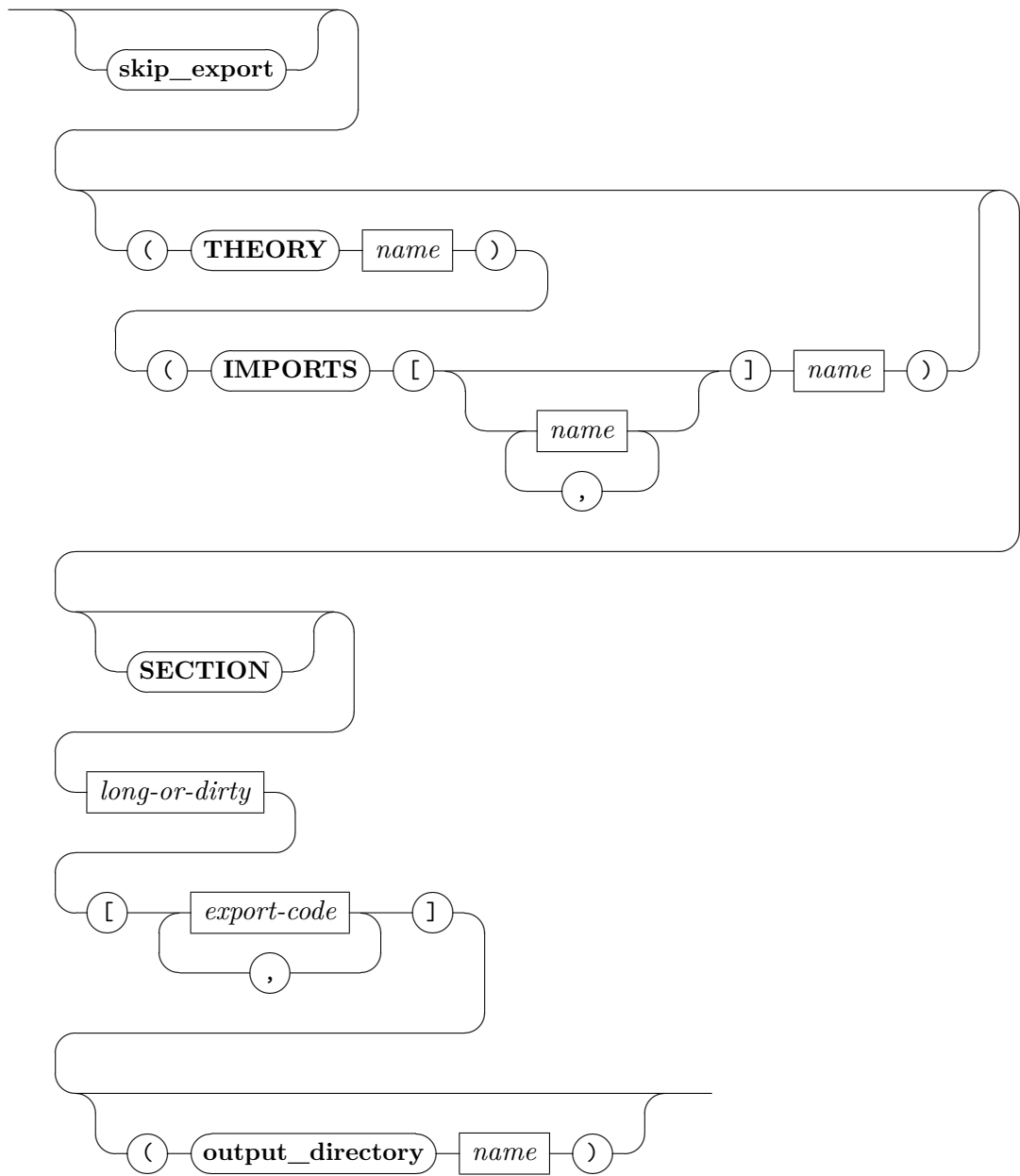
syntax



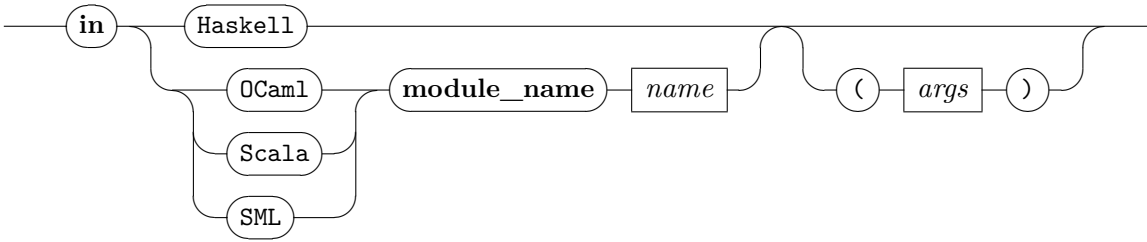
semantics



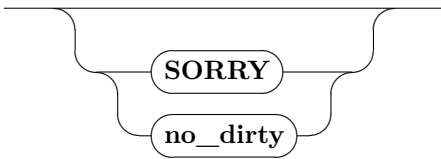
deep-embedding



export-code



long-or-dirty



generation-syntax sets the behavior of all incoming meta-commands. By default, without firstly writing **generation-syntax**, meta-commands will only print in output what they have parsed, this is similar as giving to **generation-syntax** a non-empty list having only **syntax-print** as elements (on the other hand, nothing is printed when an empty list is received). Additionally **syntax-print** can be followed by an integer indicating the printing depth in output, similar as declaring *ML-print-depth* with an integer, but the global option **syntax-print** is restricted to meta-commands. Besides the printing of syntaxes, several options are provided to further analyze the semantics of languages being embedded, and tell if their evaluation should occur immediately using the **shallow** mode, or to only display what would have been evaluated using the **deep** mode (i.e., to only show the generated Isabelle content in the output window).

Since several occurrences of **deep**, **shallow** or **syntax-print** can appear in the parameterizing list, for each meta-command the overall evaluation respects the order of events given in the list (from head to tail). At the time of writing, it is only possible to evaluate this list sequentially: the execution stops as soon as one first error is raised, thus ignoring remaining events.

generation-syntax deep flush-all performs as side effect the writing of all the generated Isabelle contents to the hard disk (all at the calling time), by iterating the saving for each **deep** mode in the list. In particular, this is only effective if there is at least one **deep** mode earlier declared.

As a side note, target languages for the **deep** mode currently supported are: Haskell, OCaml, Scala and SML. So in principle, all these targets generate the same Isabelle content and exit correctly. However, depending on the intended use, exporting with some targets may be more appropriate than other targets:

- For efficiency reasons, the meta-compiler has implemented a particular optimization for accelerating the process of evaluating incoming meta-commands. By default in Haskell and OCaml, the meta-compiler (at HOL side) is exported only

once, during the **generation-syntax** step. Then all incoming meta-commands are considered as arguments sent to the exported meta-compiler. As a compositionality aspect, these arguments are compiled then linked together with the (already compiled) meta-compiler, but this implies the use of one call of *unsafeCoerce* in Haskell and one *Obj.magic* statement in OCaml (otherwise another solution would be to extract the meta-compiler as a functor). Similar optimizations are not yet implemented for Scala and are only half-implemented for the SML target (which basically performs a step of marshalling to string in Isabelle/ML).

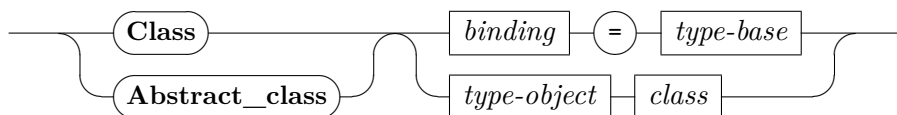
- For safety reasons, it simply suffices to extract all the meta-compiler together with the respective arguments in front of each incoming meta-commands everytime, then the overall needs to be newly compiled everytime. This is the current implemented behavior for Scala. For Haskell, OCaml and SML, it was also the default behavior in a prototyping version of the compiler, as a consequence one can restore that functionality for future versions.

Concerning the semantics of generated contents, if lemmas and proofs are generated, **SORRY** allows to explicitly skip the evaluation of all proofs, irrespective of the presence of **sorry** or not in generated proofs. In any cases, the semantics of **sorry** has not been overloaded, e.g., red background may appear as usual.

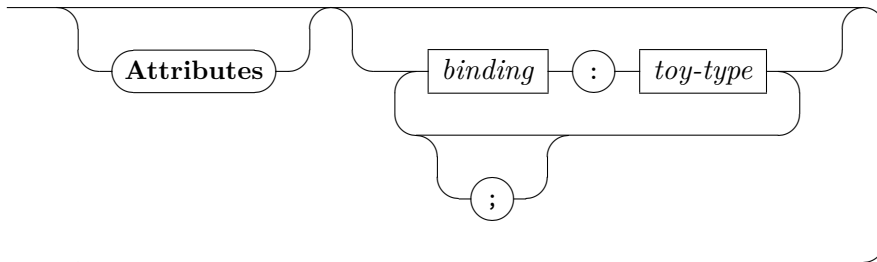
Finally **generation-semantic** is a container for specifying various options for varying the semantics of languages being embedded. For example, **design** and **analysis** are two options for specifying how the modelling of objects will be represented in the Toy Language. Similarly, this would be a typical place for options like *eager* or *lazy* for choosing how the evaluation should happen...

A.2. All Meta Commands of the Toy Language

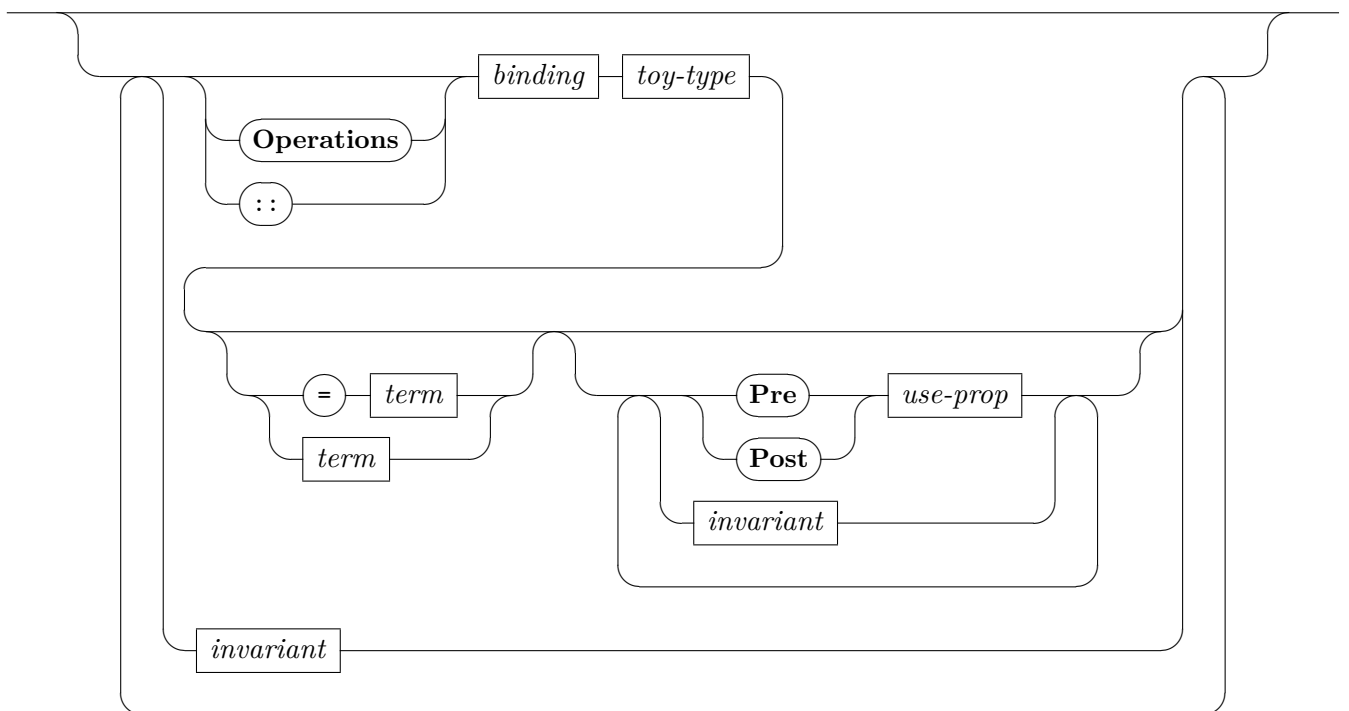
Class : *theory* → *theory*
Abstract-class : *theory* → *theory*



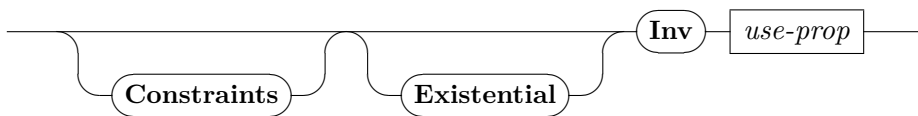
class



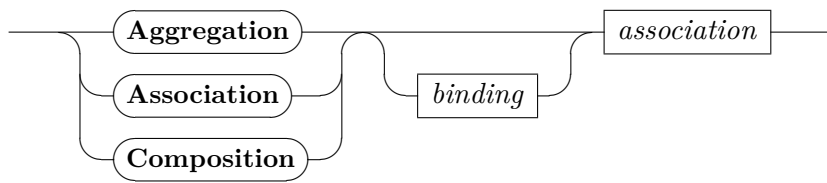
context



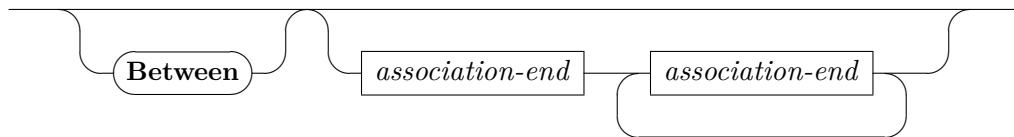
invariant



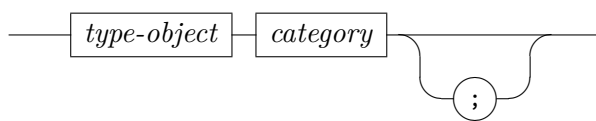
- Aggregation** : $theory \rightarrow theory$
- Association** : $theory \rightarrow theory$
- Composition** : $theory \rightarrow theory$



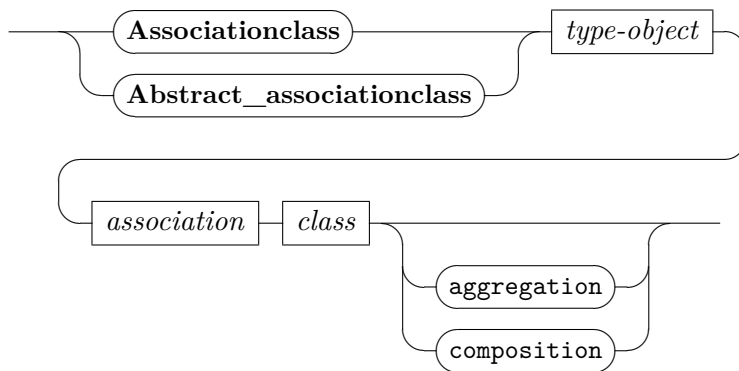
association



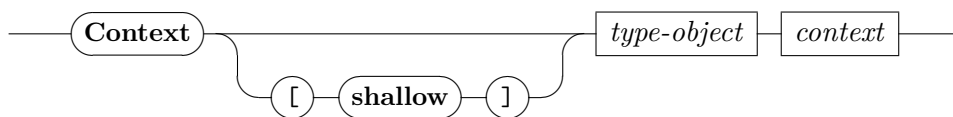
association-end



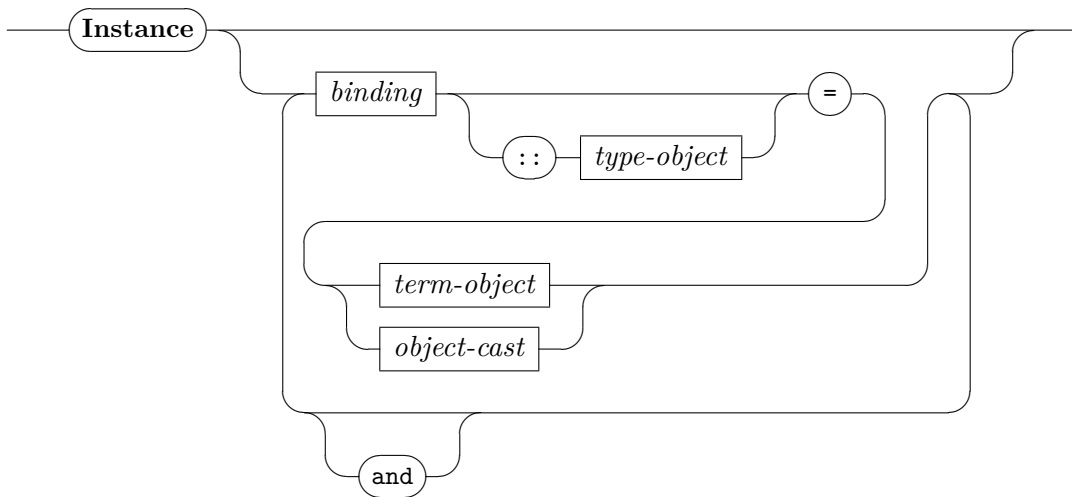
Associationclass : *theory* → *theory*
Abstract-associationclass : *theory* → *theory*



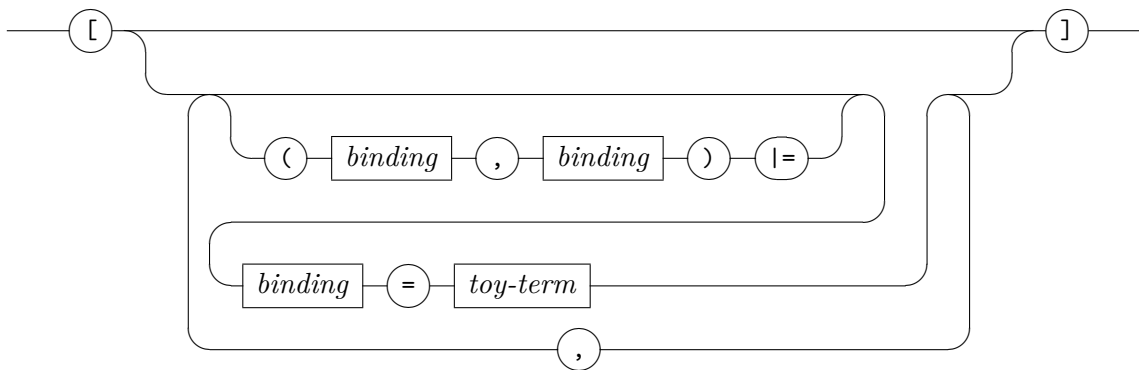
Context : *theory* → *theory*



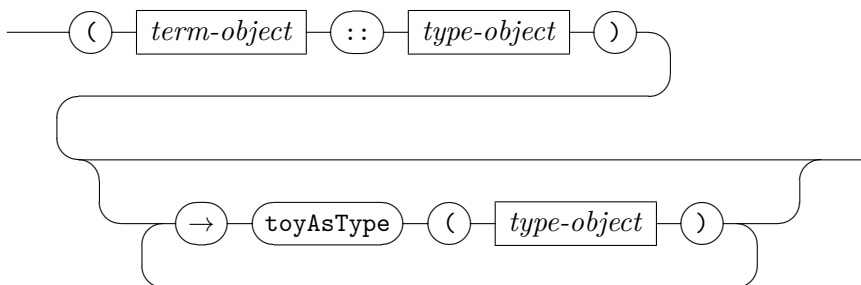
Instance : *theory* → *theory*



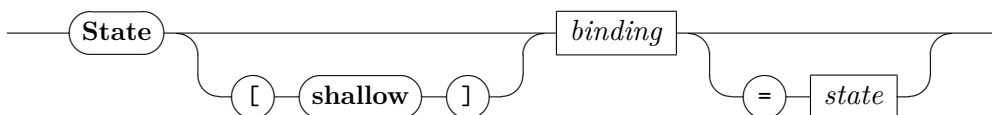
term-object



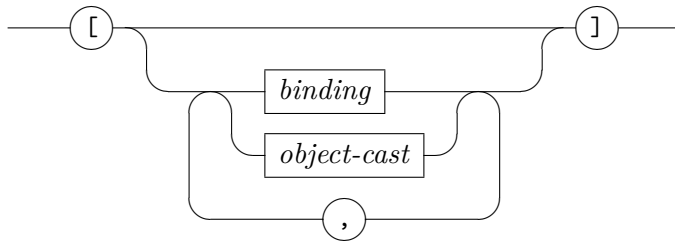
object-cast



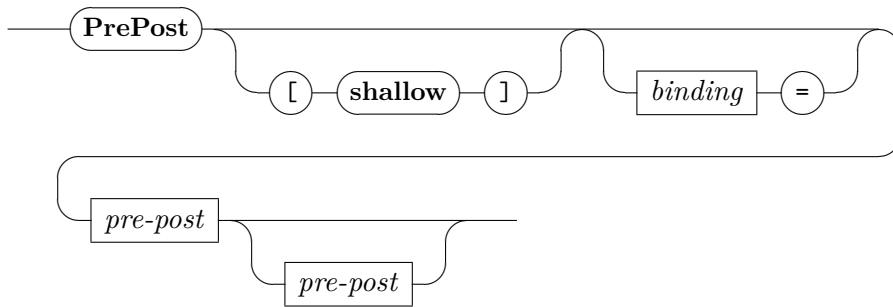
State : *theory* → *theory*



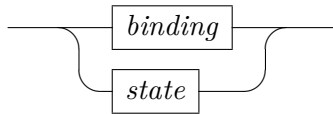
state



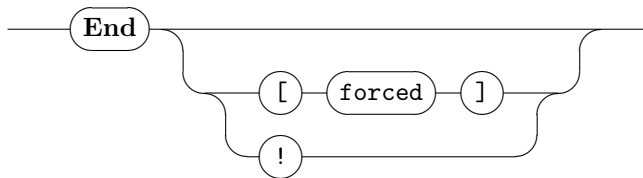
PrePost : *theory* \rightarrow *theory*



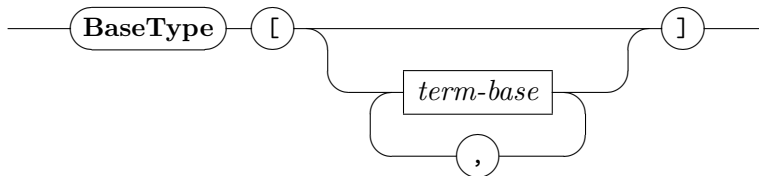
pre-post



End : *theory* \rightarrow *theory*

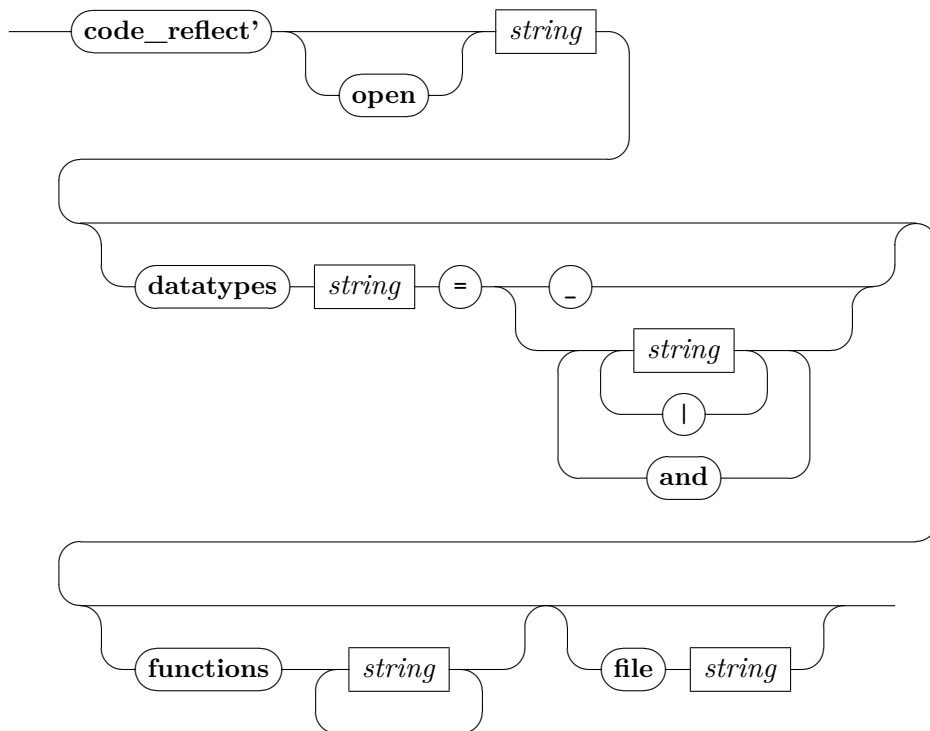


BaseType : *theory* \rightarrow *theory*



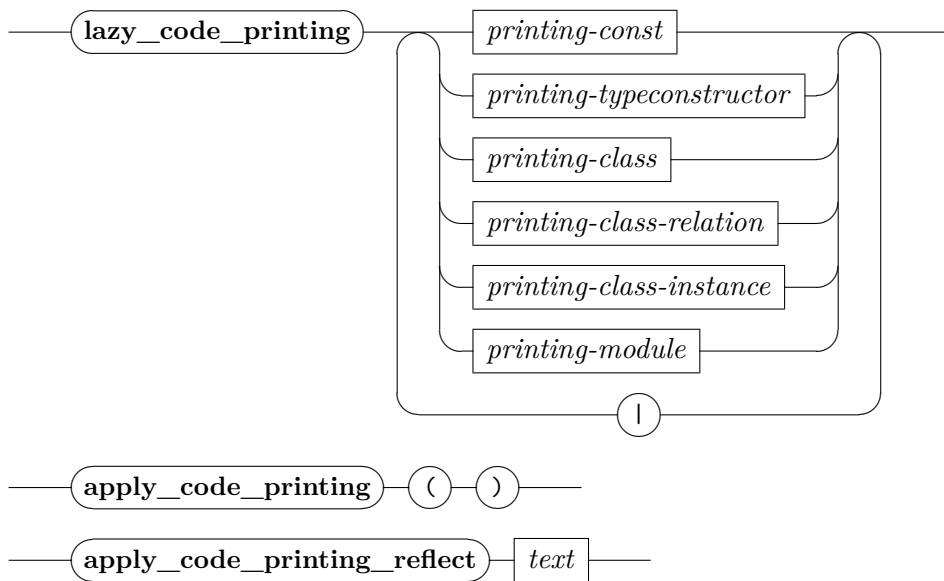
A.3. Extensions of Isabelle Commands

code-reflect' : *theory* → *theory*



code-reflect' has the same semantics as **code-reflect** except that it additionally contains the option **open** inspired from the command **export-code** (with the same semantics).

lazy-code-printing : *theory* → *theory*
apply-code-printing : *theory* → *theory*
apply-code-printing-reflect : *local-theory* → *local-theory*



lazy-code-printing has the same semantics as **code-printing** or **ML**, except that no side effects occur until we give more details about its intended future semantics: this will be precised by calling **apply-code-printing** or **apply-code-printing-reflect**.

apply-code-printing repeatedly calls **code-printing** to all previously registered elements with **lazy-code-printing** (the order is preserved).

apply-code-printing-reflect repeatedly calls **ML** to all previously registered elements with **lazy-code-printing** (the order is preserved). As a consequence, code for other targets (Haskell, OCaml, Scala) are ignored. Moreover before the execution of the overall, it is possible to give an additional piece of SML code as argument to priorly execute.

B. Content of the Directory `isabelle_home`

B.1. Extensions for Cartouches

- `./src/HOL/ex/Isabelle_Cartouche_Examples.thy` *Main0:*
Some functions have been generalized for supporting cartouches.

B.2. Other Changes

- `./src/Tools/Code/Isabelle_code_runtime.thy` *Main1:*
The option *open* was introduced in this file for the definition of *code_reflect'*.
- `./src/Tools/Code/Isabelle_code_target.thy` *Main1:*
Some signatures was removed for exposing the main structure, we have also defined at the end the implementation of *lazy_code_printing*, *apply_code_printing* and *apply_code_printing_reflect*.
- `./src/Pure/Isar/Isabelle_typedec1.thy` *Main2:*
Short modification of the argument lifting a *binding* to a *binding option* with some signatures removed.

C. Content of One Generated File (as example)

```
theory Design-generated-generated imports ../Toy-Library ../Toy-Library-Static begin
```

For certain concepts like classes and class-types, only a generic definition for its resulting semantics can be given. Generic means, there is a function outside HOL that “compiles” a concrete, closed-world class diagram into a “theory” of this data model, consisting of a bunch of definitions for classes, accessors, method, casts, and tests for actual types, as well as proofs for the fundamental properties of these operations in this concrete data model.

Our data universe consists in the concrete class diagram just of node’s, and implicitly of the class object. Each class implies the existence of a class type defined for the corresponding object representations as follows:

```
datatype ty $\mathcal{E}\mathcal{X}\mathcal{T}_{Atom}$  = mk $\mathcal{E}\mathcal{X}\mathcal{T}_{Atom}$  oid oid list option int option bool option nat option unit option
datatype ty $\mathcal{A}tom$  = mk $\mathcal{A}tom$  ty $\mathcal{E}\mathcal{X}\mathcal{T}_{Atom}$  int option
datatype ty $\mathcal{E}\mathcal{X}\mathcal{T}_{Molecule}$  = mk $\mathcal{E}\mathcal{X}\mathcal{T}_{Molecule-Atom}$  ty $\mathcal{A}tom$ 
| mk $\mathcal{E}\mathcal{X}\mathcal{T}_{Molecule}$  oid oid list option int option bool option nat option unit option
datatype ty $\mathcal{M}olecule$  = mk $\mathcal{M}olecule$  ty $\mathcal{E}\mathcal{X}\mathcal{T}_{Molecule}$ 
datatype ty $\mathcal{E}\mathcal{X}\mathcal{T}_{Person}$  = mk $\mathcal{E}\mathcal{X}\mathcal{T}_{Person-Molecule}$  ty $\mathcal{M}olecule$ 
| mk $\mathcal{E}\mathcal{X}\mathcal{T}_{Person-Atom}$  ty $\mathcal{A}tom$ 
| mk $\mathcal{E}\mathcal{X}\mathcal{T}_{Person}$  oid nat option unit option
datatype ty $\mathcal{P}erson$  = mk $\mathcal{P}erson$  ty $\mathcal{E}\mathcal{X}\mathcal{T}_{Person}$  oid list option int option bool option
datatype ty $\mathcal{E}\mathcal{X}\mathcal{T}_{Galaxy}$  = mk $\mathcal{E}\mathcal{X}\mathcal{T}_{Galaxy-Person}$  ty $\mathcal{P}erson$ 
| mk $\mathcal{E}\mathcal{X}\mathcal{T}_{Galaxy-Molecule}$  ty $\mathcal{M}olecule$ 
| mk $\mathcal{E}\mathcal{X}\mathcal{T}_{Galaxy-Atom}$  ty $\mathcal{A}tom$ 
| mk $\mathcal{E}\mathcal{X}\mathcal{T}_{Galaxy}$  oid
datatype ty $\mathcal{G}alaxy$  = mk $\mathcal{G}alaxy$  ty $\mathcal{E}\mathcal{X}\mathcal{T}_{Galaxy}$  nat option unit option
datatype ty $\mathcal{E}\mathcal{X}\mathcal{T}_{ToyAny}$  = mk $\mathcal{E}\mathcal{X}\mathcal{T}_{ToyAny-Galaxy}$  ty $\mathcal{G}alaxy$ 
| mk $\mathcal{E}\mathcal{X}\mathcal{T}_{ToyAny-Person}$  ty $\mathcal{P}erson$ 
| mk $\mathcal{E}\mathcal{X}\mathcal{T}_{ToyAny-Molecule}$  ty $\mathcal{M}olecule$ 
| mk $\mathcal{E}\mathcal{X}\mathcal{T}_{ToyAny-Atom}$  ty $\mathcal{A}tom$ 
| mk $\mathcal{E}\mathcal{X}\mathcal{T}_{ToyAny}$  oid
datatype ty $\mathcal{T}oyAny$  = mk $\mathcal{T}oyAny$  ty $\mathcal{E}\mathcal{X}\mathcal{T}_{ToyAny}$ 
```

Now, we construct a concrete “universe of ToyAny types” by injection into a sum type containing the class types. This type of ToyAny will be used as instance for all respective type-variables.

```

datatype  $\mathfrak{A}$  = inAtom tyAtom
                | inMolecule tyMolecule
                | inPerson tyPerson
                | inGalaxy tyGalaxy
                | inToyAny tyToyAny

```

Having fixed the object universe, we can introduce type synonyms that exactly correspond to Toy types. Again, we exploit that our representation of Toy is a “shallow embedding” with a one-to-one correspondance of Toy-types to types of the meta-language HOL.

```

type-synonym Atom =  $\langle\langle ty_{Atom} \rangle_{\perp}\rangle_{\perp}$ 
type-synonym Molecule =  $\langle\langle ty_{Molecule} \rangle_{\perp}\rangle_{\perp}$ 
type-synonym Person =  $\langle\langle ty_{Person} \rangle_{\perp}\rangle_{\perp}$ 
type-synonym Galaxy =  $\langle\langle ty_{Galaxy} \rangle_{\perp}\rangle_{\perp}$ 
type-synonym ToyAny =  $\langle\langle ty_{ToyAny} \rangle_{\perp}\rangle_{\perp}$ 

```

definition *oid_{Atom}-0--boss* = 0

definition *oid_{Molecule}-0--boss* = 0

definition *oid_{Person}-0--boss* = 0

definition *switch₂-01* = $(\lambda [x0 , x1] \Rightarrow (x0 , x1))$

definition *switch₂-10* = $(\lambda [x0 , x1] \Rightarrow (x1 , x0))$

definition *oid1* = 1

definition *oid2* = 2

definition *inst-assoc1* = $(\lambda oid-class\ to-from\ oid.\ ((case\ (deref-assocs-list\ ((to-from::oid\ list\ list\ \Rightarrow\ oid\ list\ \times\ oid\ list))\ ((oid::oid))\ ((drop\ (((map-of-list\ ([oid_{Person}-0--boss , (List.map\ ((\lambda(x , y).\ [x , y])\ o\ switch_{2-01})\ ([[[oid1] , [oid2]]])))\ (((oid-class::oid))))))\ of\ Nil\ \Rightarrow\ None\ | l \Rightarrow (Some\ (l))::oid\ list\ option))$

definition *oid3* = 3

definition *inst-assoc3* = $(\lambda oid-class\ to-from\ oid.\ ((case\ (deref-assocs-list\ ((to-from::oid\ list\ list\ \Rightarrow\ oid\ list\ \times\ oid\ list))\ ((oid::oid))\ ((drop\ (((map-of-list\ ([[]])\ ((oid-class::oid))))))\ of\ Nil\ \Rightarrow\ None\ | l \Rightarrow (Some\ (l))::oid\ list\ option))$

definition *oid4* = 4

definition *oid5* = 5

definition *oid6* = 6

definition *oid7* = 7

definition $inst\text{-}assoc_4 = (\lambda oid\text{-}class\ to\text{-}from\ oid. ((case\ (deref\text{-}assoc\text{-}list\ ((to\text{-}from::oid\ list\ list\ \Rightarrow\ oid\ list\ \times\ oid\ list))\ ((oid::oid)\ ((drop\ (((map\text{-}of\text{-}list\ ((oid_{Person}\text{-}0\text{-}boss\ ,\ (List.map\ ((\lambda(x\ ,\ y).\ [x\ ,\ y])\ o\ switch_2\text{-}01)\ ([[oid7]\ ,\ [oid6]]\ ,\ [[oid6]\ ,\ [oid1]]\ ,\ [[oid4]\ ,\ [oid5]]))))))))\ ((oid\text{-}class::oid))))))\ of\ Nil\ \Rightarrow\ None\ | l\ \Rightarrow\ (Some\ (l))::oid\ list\ option))$

locale $state\text{-}\sigma_1 =$
fixes $oid_4 :: nat$
fixes $oid_5 :: nat$
fixes $oid_6 :: nat$
fixes $oid_1 :: nat$
fixes $oid_7 :: nat$
fixes $oid_2 :: nat$
assumes $distinct\text{-}oid: (distinct\ ([oid_4\ ,\ oid_5\ ,\ oid_6\ ,\ oid_1\ ,\ oid_7\ ,\ oid_2]))$
fixes $\sigma_1\text{-}object0_{Person} :: ty_{Person}$
fixes $\sigma_1\text{-}object0 :: \cdot Person$
assumes $\sigma_1\text{-}object0\text{-}def: \sigma_1\text{-}object0 = (\lambda\cdot. \llbracket \sigma_1\text{-}object0_{Person} \rrbracket)$
fixes $\sigma_1\text{-}object1_{Person} :: ty_{Person}$
fixes $\sigma_1\text{-}object1 :: \cdot Person$
assumes $\sigma_1\text{-}object1\text{-}def: \sigma_1\text{-}object1 = (\lambda\cdot. \llbracket \sigma_1\text{-}object1_{Person} \rrbracket)$
fixes $\sigma_1\text{-}object2_{Person} :: ty_{Person}$
fixes $\sigma_1\text{-}object2 :: \cdot Person$
assumes $\sigma_1\text{-}object2\text{-}def: \sigma_1\text{-}object2 = (\lambda\cdot. \llbracket \sigma_1\text{-}object2_{Person} \rrbracket)$
fixes $X_{Person}1_{Person} :: ty_{Person}$
fixes $X_{Person}1 :: \cdot Person$
assumes $X_{Person}1\text{-}def: X_{Person}1 = (\lambda\cdot. \llbracket X_{Person}1_{Person} \rrbracket)$
fixes $\sigma_1\text{-}object4_{Person} :: ty_{Person}$
fixes $\sigma_1\text{-}object4 :: \cdot Person$
assumes $\sigma_1\text{-}object4\text{-}def: \sigma_1\text{-}object4 = (\lambda\cdot. \llbracket \sigma_1\text{-}object4_{Person} \rrbracket)$
fixes $X_{Person}2_{Person} :: ty_{Person}$
fixes $X_{Person}2 :: \cdot Person$
assumes $X_{Person}2\text{-}def: X_{Person}2 = (\lambda\cdot. \llbracket X_{Person}2_{Person} \rrbracket)$
begin
definition $\sigma_1 = (state.make\ ((Map.empty\ (oid_4\ \mapsto\ (in_{Person}\ (\sigma_1\text{-}object0_{Person})),\ oid_5\ \mapsto\ (in_{Person}\ (\sigma_1\text{-}object1_{Person})),\ oid_6\ \mapsto\ (in_{Person}\ (\sigma_1\text{-}object2_{Person})),\ oid_1\ \mapsto\ (in_{Person}\ (X_{Person}1_{Person})),\ oid_7\ \mapsto\ (in_{Person}\ (\sigma_1\text{-}object4_{Person})),\ oid_2\ \mapsto\ (in_{Person}\ (X_{Person}2_{Person}))))\ ((map\text{-}of\text{-}list\ ((oid_{Person}\text{-}0\text{-}boss\ ,\ (List.map\ ((\lambda(x\ ,\ y).\ [x\ ,\ y])\ o\ switch_2\text{-}01)\ ([[oid_4]\ ,\ [oid_2]]\ ,\ [[oid_6]\ ,\ [oid_4]]\ ,\ [[oid_1]\ ,\ [oid_6]]\ ,\ [[oid_7]\ ,\ [oid_3]]))))))))$

lemma $perm\text{-}\sigma_1 : \sigma_1 = (state.make\ ((Map.empty\ (oid_2\ \mapsto\ (in_{Person}\ (X_{Person}2_{Person})),\ oid_7\ \mapsto\ (in_{Person}\ (\sigma_1\text{-}object4_{Person})),\ oid_1\ \mapsto\ (in_{Person}\ (X_{Person}1_{Person})),\ oid_6\ \mapsto\ (in_{Person}\ (\sigma_1\text{-}object2_{Person})),\ oid_5\ \mapsto\ (in_{Person}\ (\sigma_1\text{-}object1_{Person})),\ oid_4\ \mapsto\ (in_{Person}\ (\sigma_1\text{-}object0_{Person}))))\ ((assoc\ (\sigma_1))))$
 $\langle proof \rangle$
end

```

locale state- $\sigma_1'$  =
fixes oid1 :: nat
fixes oid2 :: nat
fixes oid3 :: nat
assumes distinct-oid: (distinct ([oid1 , oid2 , oid3]))
fixes XPerson1Person :: tyPerson
fixes XPerson1 :: Person
assumes XPerson1-def: XPerson1 = ( $\lambda$ -. [[XPerson1Person]])
fixes XPerson2Person :: tyPerson
fixes XPerson2 :: Person
assumes XPerson2-def: XPerson2 = ( $\lambda$ -. [[XPerson2Person]])
fixes XPerson3Person :: tyPerson
fixes XPerson3 :: Person
assumes XPerson3-def: XPerson3 = ( $\lambda$ -. [[XPerson3Person]])
begin
definition  $\sigma_1' = (\text{state.make } ((\text{Map.empty } (oid1 \mapsto (\text{in}_{Person} (X_{Person1Person})), oid2 \mapsto$ 
(inPerson (XPerson2Person)), oid3  $\mapsto$  (inPerson (XPerson3Person)))))) ((map-of-list ((oidPerson-0---boss
, (List.map (( $\lambda(x, y).$  [x , y]) o switch2-01) ([[[oid1 ] , [oid2]]])))))))
lemma perm- $\sigma_1'$ :  $\sigma_1' = (\text{state.make } ((\text{Map.empty } (oid3 \mapsto (\text{in}_{Person} (X_{Person3Person})), oid2 \mapsto$ 
(inPerson (XPerson2Person)), oid1  $\mapsto$  (inPerson (XPerson1Person)))))) ((assocs ( $\sigma_1'$ )))
  <proof>
end

```

```

locale pre-post- $\sigma_1$ - $\sigma_1'$  =
fixes oid1 :: nat
fixes oid2 :: nat
fixes oid3 :: nat
fixes oid4 :: nat
fixes oid5 :: nat
fixes oid6 :: nat
fixes oid7 :: nat
assumes distinct-oid: (distinct ([oid1 , oid2 , oid3 , oid4 , oid5 , oid6 , oid7]))
fixes XPerson1Person :: tyPerson
fixes XPerson1 :: Person
assumes XPerson1-def: XPerson1 = ( $\lambda$ -. [[XPerson1Person]])
fixes XPerson2Person :: tyPerson
fixes XPerson2 :: Person
assumes XPerson2-def: XPerson2 = ( $\lambda$ -. [[XPerson2Person]])
fixes XPerson3Person :: tyPerson
fixes XPerson3 :: Person
assumes XPerson3-def: XPerson3 = ( $\lambda$ -. [[XPerson3Person]])
fixes  $\sigma_1$ -object0Person :: tyPerson
fixes  $\sigma_1$ -object0 :: Person
assumes  $\sigma_1$ -object0-def:  $\sigma_1$ -object0 = ( $\lambda$ -. [[ $\sigma_1$ -object0Person]])
fixes  $\sigma_1$ -object1Person :: tyPerson
fixes  $\sigma_1$ -object1 :: Person

```

assumes $\sigma_1\text{-object1-def}$: $\sigma_1\text{-object1} = (\lambda\cdot. \llbracket \sigma_1\text{-object1}_{Person} \rrbracket)$

fixes $\sigma_1\text{-object2}_{Person} :: ty_{Person}$

fixes $\sigma_1\text{-object2} :: \cdot Person$

assumes $\sigma_1\text{-object2-def}$: $\sigma_1\text{-object2} = (\lambda\cdot. \llbracket \sigma_1\text{-object2}_{Person} \rrbracket)$

fixes $\sigma_1\text{-object4}_{Person} :: ty_{Person}$

fixes $\sigma_1\text{-object4} :: \cdot Person$

assumes $\sigma_1\text{-object4-def}$: $\sigma_1\text{-object4} = (\lambda\cdot. \llbracket \sigma_1\text{-object4}_{Person} \rrbracket)$

assumes σ_1 : $(state\text{-}\sigma_1 (oid4) (oid5) (oid6) (oid1) (oid7) (oid2) (\sigma_1\text{-object0}_{Person}) (\sigma_1\text{-object0})$
 $(\sigma_1\text{-object1}_{Person}) (\sigma_1\text{-object1}) (\sigma_1\text{-object2}_{Person}) (\sigma_1\text{-object2}) (X_{Person1}_{Person}) (X_{Person1})$
 $(\sigma_1\text{-object4}_{Person}) (\sigma_1\text{-object4}) (X_{Person2}_{Person}) (X_{Person2}))$

assumes σ_1' : $(state\text{-}\sigma_1' (oid1) (oid2) (oid3) (X_{Person1}_{Person}) (X_{Person1}) (X_{Person2}_{Person})$
 $(X_{Person2}) (X_{Person3}_{Person}) (X_{Person3}))$

begin

interpretation $state\text{-}\sigma_1$: $state\text{-}\sigma_1 \ oid4 \ oid5 \ oid6 \ oid1 \ oid7 \ oid2 \ \sigma_1\text{-object0}_{Person} \ \sigma_1\text{-object0}$
 $\sigma_1\text{-object1}_{Person} \ \sigma_1\text{-object1} \ \sigma_1\text{-object2}_{Person} \ \sigma_1\text{-object2} \ X_{Person1}_{Person} \ X_{Person1} \ \sigma_1\text{-object4}_{Person}$
 $\sigma_1\text{-object4} \ X_{Person2}_{Person} \ X_{Person2}$
 $\langle proof \rangle$

interpretation $state\text{-}\sigma_1'$: $state\text{-}\sigma_1' \ oid1 \ oid2 \ oid3 \ X_{Person1}_{Person} \ X_{Person1} \ X_{Person2}_{Person}$
 $X_{Person2} \ X_{Person3}_{Person} \ X_{Person3}$
 $\langle proof \rangle$

definition $heap\text{-}\sigma_1 = (heap (state\text{-}\sigma_1.\sigma_1))$

definition $heap\text{-}\sigma_1' = (heap (state\text{-}\sigma_1'.\sigma_1'))$

end

end