

# Faithful Logic Embeddings in HOL — Deep and Shallow (Isabelle/HOL dataset)

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## Abstract

A recipe for the simultaneous deployment of different forms of deep and shallow embeddings of non-classical logics in classical higher-order logic is presented, which enables interactive or even automated faithfulness proofs between the logic embeddings. The approach, which is particularly fruitful for logic education, is explained in detail in an associated CADE conference paper. This paper presents the corresponding Isabelle/HOL dataset (which is only slightly modified to meet AFP requirements).

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## 1 Introduction

The Isabelle/HOL dataset associated with [1] is presented. Sections 3, 4 and 5 present deep, maximally shallow, and minimally shallow embeddings of propositional modal logic (PML) in classical higher-order logic (HOL). These are connected, as a novel contribution, by automated faithfulness proofs given in Sect. 6. This connection ensures that these deep and shallow embeddings can now be used interchangeably in subsequent applications. Several experiments with the presented embeddings are presented in Sect. 7. The presented work is conceptual in nature and can be adapted to other non-classical logics. For more detailed explanations of the presented material, including a discussion of related works, see [1].

## 2 Preliminaries

The following preliminaries are shared between all embeddings introduced in the remainder of this paper.

```
theory PMLinHOL-preliminaries
  imports Main
begin
```

— Type declarations common for both the deep and shallow embedding

**typedecl**  $w$  — Type for possible worlds

**typedecl**  $\mathcal{S}$  — Type for propositional constant symbols

**consts**  $p::\mathcal{S}$   $q::\mathcal{S}$  — Some propositional constant symbols

**type-synonym**  $\mathcal{W} = w \Rightarrow bool$  — Type for sets of possible worlds

**type-synonym**  $\mathcal{R} = w \Rightarrow w \Rightarrow bool$  — Type for accessibility relations

**type-synonym**  $\mathcal{V} = \mathcal{S} \Rightarrow w \Rightarrow bool$  — Type for valuation functions

— Some useful predicates for accessibility relations

**abbreviation**(*input*) *reflexive*  $\equiv \lambda R::\mathcal{R}. \forall x. R\ x\ x$

**abbreviation**(*input*) *symmetric*  $\equiv \lambda R::\mathcal{R}. \forall x\ y. R\ x\ y \longrightarrow R\ y\ x$

**abbreviation**(*input*) *transitive*  $\equiv \lambda R::\mathcal{R}. \forall x\ y\ z. (R\ x\ y \wedge R\ y\ z) \longrightarrow R\ x\ z$

**abbreviation**(*input*) *equivrel*  $\equiv \lambda R::\mathcal{R}. reflexive\ R \wedge symmetric\ R \wedge transitive\ R$

**abbreviation**(*input*) *irreflexive*  $\equiv \lambda R::\mathcal{R}. \forall x. \neg R\ x\ x$

**abbreviation**(*input*) *euclidean*  $\equiv \lambda R::\mathcal{R}. \forall x\ y\ z. R\ x\ y \wedge R\ x\ z \longrightarrow R\ y\ z$

**abbreviation**(*input*) *wellfounded*  $\equiv \lambda R::\mathcal{R}. \forall P::\mathcal{W}. (\forall x. (\forall y. R\ y\ x \longrightarrow P\ y) \longrightarrow P\ x) \longrightarrow (\forall x. P\ x)$

**abbreviation**(*input*) *converserel*  $\equiv \lambda R::\mathcal{R}. \lambda y::w. \lambda x::w. R\ x\ y$

**abbreviation**(*input*) *conversewf*  $\equiv \lambda R::\mathcal{R}. \text{wellfounded } (\text{converserel } R)$

— Bounded universal quantifier:  $\forall x:W. \varphi$  stands for  $\forall x. W x \longrightarrow \varphi x$

**abbreviation**(*input*) *BoundedAll*:: $\mathcal{W}\Rightarrow\mathcal{W}\Rightarrow\text{bool}$  **where** *BoundedAll*  $W \varphi \equiv \forall x. W x \longrightarrow \varphi x$

**syntax** *-BoundedAll*:: $\text{pttrn}\Rightarrow\mathcal{W}\Rightarrow\text{bool}\Rightarrow\text{bool}$   $((\exists\forall(-/:-)/-)$   $[0, 0, 10]$   $10)$

**translations**  $\forall x:W. \varphi \equiv \text{CONST } \text{BoundedAll } W (\lambda x. \varphi)$

— Backward implication; useful for aesthetic reasons

**abbreviation**(*input*) *Bimp* (**infixr**  $\longleftarrow$  50) **where**  $\varphi \longleftarrow \psi \equiv \psi \longrightarrow \varphi$

— Some further settings

**declare** $[[\text{syntax-ambiguity-warning}=\text{false}]]$

**nitpick-params** $[\text{user-axioms}, \text{expect}=\text{genuine}, \text{timeout}=60]$

**end**

### 3 Deep embedding of PML in HOL

**theory** *PMLinHOL-deep*

**imports** *PMLinHOL-preliminaries*

**begin**

— Deep embedding (of propositional modal logic in HOL)

**datatype** *PML* = *AtmD*  $\mathcal{S}$   $(-^d)$  | *NotD* *PML*  $(-^d)$  | *ImpD* *PML* *PML* (**infixr**  $\supset^d$  93) | *BoxD* *PML*  $(\Box^d)$

— Further logical connectives as definitions

**definition** *OrD* (**infixr**  $\vee^d$  92) **where**  $\varphi \vee^d \psi \equiv \neg^d \varphi \supset^d \psi$

**definition** *AndD* (**infixr**  $\wedge^d$  95) **where**  $\varphi \wedge^d \psi \equiv \neg^d (\varphi \supset^d \neg^d \psi)$

**definition** *DiaD*  $(\diamond^d)$  **where**  $\diamond^d \varphi \equiv \neg^d (\Box^d (\neg^d \varphi))$

**definition** *TopD*  $(\top^d)$  **where**  $\top^d \equiv p^d \supset^d p^d$

**definition** *BotD*  $(\perp^d)$  **where**  $\perp^d \equiv \neg^d \top^d$

— Definition of truth of a formula relative to a model  $\langle W, R, V \rangle$  and possible world  $w$

**primrec** *RelativeTruthD* ::  $\mathcal{W}\Rightarrow\mathcal{R}\Rightarrow\mathcal{V}\Rightarrow w\Rightarrow\text{PML}\Rightarrow\text{bool}$   $(\langle -, -, - \rangle, - \models^d -)$  **where**

$\langle W, R, V \rangle, w \models^d a^d = (V a w)$

|  $\langle W, R, V \rangle, w \models^d \neg^d \varphi = (\neg \langle W, R, V \rangle, w \models^d \varphi)$

|  $\langle W, R, V \rangle, w \models^d \varphi \supset^d \psi = (\langle W, R, V \rangle, w \models^d \varphi \longrightarrow \langle W, R, V \rangle, w \models^d \psi)$

|  $\langle W, R, V \rangle, w \models^d \Box^d \varphi = (\forall v:W. R w v \longrightarrow \langle W, R, V \rangle, v \models^d \varphi)$

— Definition of validity

**definition** *ValD*  $(\models^d -)$  **where**  $(\models^d \varphi) \equiv (\forall W R V. \forall w:W. \langle W, R, V \rangle, w \models^d \varphi)$

— Collection of definitions in a bag called DefD

**named-theorems** *DefD* **declare** *OrD-def* $[\text{DefD}, \text{simp}]$  *AndD-def* $[\text{DefD}, \text{simp}]$  *DiaD-def* $[\text{DefD}, \text{simp}]$  *TopD-def* $[\text{DefD}, \text{simp}]$  *BotD-def* $[\text{DefD}, \text{simp}]$  *RelativeTruthD-def* $[\text{DefD}, \text{simp}]$  *ValD-def* $[\text{DefD}, \text{simp}]$

**end**

## 4 Shallow embedding of PML in HOL (maximal)

**theory** *PMLinHOL-shallow*

**imports** *PMLinHOL-preliminaries*

**begin**

— Shallow embedding (of propositional modal logic in HOL)

**type-synonym**  $\sigma = \mathcal{W} \Rightarrow \mathcal{R} \Rightarrow \mathcal{V} \Rightarrow w \Rightarrow bool$

**definition** *AtmS*:: $\mathcal{S} \Rightarrow \sigma$  ( $-^s$ ) **where**  $a^s \equiv \lambda W R V w. V a w$

**definition** *NegS*:: $\sigma \Rightarrow \sigma$  ( $\neg^s$ ) **where**  $\neg^s \varphi \equiv \lambda W R V w. \neg(\varphi W R V w)$

**definition** *ImpS*:: $\sigma \Rightarrow \sigma \Rightarrow \sigma$  (**infix**  $\supset^s$  93) **where**  $\varphi \supset^s \psi \equiv \lambda W R V w. (\varphi W R V w) \longrightarrow (\psi W R V w)$

**definition** *BoxS*:: $\sigma \Rightarrow \sigma$  ( $\Box^s$ ) **where**  $\Box^s \varphi \equiv \lambda W R V w. \forall v:W. R w v \longrightarrow (\varphi W R V v)$

— Further logical connectives as definitions

**definition** *OrS* (**infix**  $\vee^s$  92) **where**  $\varphi \vee^s \psi \equiv \neg^s \varphi \supset^s \psi$

**definition** *AndS* (**infix**  $\wedge^s$  95) **where**  $\varphi \wedge^s \psi \equiv \neg^s(\varphi \supset^s \neg^s \psi)$

**definition** *DiaS* ( $\Diamond^s$ ) **where**  $\Diamond^s \varphi \equiv \neg^s(\Box^s(\neg^s \varphi))$

**definition** *TopS* ( $\top^s$ ) **where**  $\top^s \equiv p^s \supset^s p^s$

**definition** *BotS* ( $\perp^s$ ) **where**  $\perp^s \equiv \neg^s \top^s$

— Definition of truth of a formula relative to a model  $\langle W, R, V \rangle$  and possible world  $w$

**definition** *RelativeTruthS*:: $\mathcal{W} \Rightarrow \mathcal{R} \Rightarrow \mathcal{V} \Rightarrow w \Rightarrow \sigma \Rightarrow bool$  ( $\langle -, -, - \rangle, \models^s -$ ) **where**  $\langle W, R, V \rangle, w \models^s \varphi \equiv \varphi W R V w$

— Definition of validity

**definition** *ValS* ( $\models^s -$ ) **where**  $\models^s \varphi \equiv \forall W R V. \forall w:W. \langle W, R, V \rangle, w \models^s \varphi$

— Collection of definitions in a bag called DefS

**named-theorems** *DefS* **declare** *AtmS-def*[*DefS, simp*] *NegS-def*[*DefS, simp*] *ImpS-def*[*DefS, simp*] *BoxS-def*[*DefS, simp*] *OrS-def*[*DefS, simp*] *AndS-def*[*DefS, simp*] *DiaS-def*[*DefS, simp*] *TopS-def*[*DefS, simp*] *BotS-def*[*DefS, simp*] *RelativeTruthS-def*[*DefS, simp*] *ValS-def*[*DefS, simp*]  
**end**

## 5 Shallow embedding of PML in HOL (minimal)

**theory** *PMLinHOL-shallow-minimal*

**imports** *PMLinHOL-preliminaries*

**begin**

— The accessibility relation  $R$  and the valuation function  $V$  are introduced as constants at the meta-level HOL

**consts** *R*:: $\mathcal{R}$  *V*:: $\mathcal{V}$

— Shallow embedding (of propositional modal logic in HOL)

**type-synonym**  $\sigma = w \Rightarrow bool$

**definition** *AtmM*:: $\mathcal{S} \Rightarrow \sigma$  ( $-^m$ ) **where**  $a^m \equiv \lambda w. V a w$

**definition**  $NegM::\sigma\Rightarrow\sigma$  ( $\neg^m$ ) **where**  $\neg^m \varphi \equiv \lambda w. \neg \varphi w$   
**definition**  $ImpM::\sigma\Rightarrow\sigma\Rightarrow\sigma$  (**infixr**  $\supset^m$  93) **where**  $\varphi \supset^m \psi \equiv \lambda w. \varphi w \longrightarrow \psi w$   
**definition**  $BoxM::\sigma\Rightarrow\sigma$  ( $\Box^m$ ) **where**  $\Box^m \varphi \equiv \lambda w. \forall v. R w v \longrightarrow \varphi v$

— Further logical connectives as definitions

**definition**  $OrM$  (**infixr**  $\vee^m$  92) **where**  $\varphi \vee^m \psi \equiv \neg^m \varphi \supset^m \psi$   
**definition**  $AndM$  (**infixr**  $\wedge^m$  95) **where**  $\varphi \wedge^m \psi \equiv \neg^m (\varphi \supset^m \neg^m \psi)$   
**definition**  $DiaM$  ( $\Diamond^m$ -) **where**  $\Diamond^m \varphi \equiv \neg^m (\Box^m (\neg^m \varphi))$   
**definition**  $TopM$  ( $\top^m$ ) **where**  $\top^m \equiv p^m \supset^m p^m$   
**definition**  $BotM$  ( $\perp^m$ ) **where**  $\perp^m \equiv \neg^m \top^m$

— Definition of truth of a formula relative to a model  $\langle W, R, V \rangle$  and a possible world  $w$

**definition**  $RelativeTruthM::w\Rightarrow\sigma\Rightarrow bool$  ( $\models^m$  -) **where**  $w \models^m \varphi \equiv \varphi w$

— Definition of validity

**definition**  $ValM$  ( $\models^m$  -) **where**  $\models^m \varphi \equiv \forall w::w. w \models^m \varphi$

— Collection of definitions in a bag called DefM

**named-theorems**  $DefM$  **declare**  $AtmM-def[DefM, simp]$   $NegM-def[DefM, simp]$   
 $ImpM-def[DefM, simp]$   $BoxM-def[DefM, simp]$   $OrM-def[DefM, simp]$   $AndM-def[DefM, simp]$   
 $DiaM-def[DefM, simp]$   $TopM-def[DefM, simp]$   $BotM-def[DefM, simp]$   $RelativeTruthM-def[DefM, simp]$   
 $ValM-def[DefM, simp]$   
**end**

## 6 Automated faithfulness proofs

**theory**  $PMLinHOL-faithfulness$

**imports**  $PMLinHOL-deep$   $PMLinHOL-shallow$   $PMLinHOL-shallow-minimal$   
**begin**

— Mappings: deep to maximal shallow and deep to minimal shallow

**primrec**  $DpToShMax$  ( $\Downarrow$ -) **where**  $\Downarrow \varphi^d = \varphi^s \mid \Downarrow \neg^d \varphi = \neg^s \Downarrow \varphi \mid \Downarrow (\varphi \supset^d \psi) = \Downarrow \varphi \supset^s \Downarrow \psi \mid \Downarrow \Box^d \varphi = \Box^s \Downarrow \varphi$   
**primrec**  $DpToShMin$  ( $\Downarrow$ -) **where**  $\Downarrow \varphi^d = \varphi^m \mid \Downarrow \neg^d \varphi = \neg^m \Downarrow \varphi \mid \Downarrow (\varphi \supset^d \psi) = \Downarrow \varphi \supset^m \Downarrow \psi \mid \Downarrow \Box^d \varphi = \Box^m \Downarrow \varphi$

— Proving faithfulness between deep and maximal shallow

**theorem**  $Faithful1a$ :  $\forall W R V. \forall w:W. \langle W, R, V \rangle, w \models^d \varphi \longleftrightarrow \langle W, R, V \rangle, w \models^s \Downarrow \varphi$  **apply** *induct by auto*  
**theorem**  $Faithful1b$ :  $\models^d \varphi \longleftrightarrow \models^s \Downarrow \varphi$  **using**  $Faithful1a$  **by** *auto*

— Proving faithfulness between deep and minimal shallow

**theorem**  $Faithful2$ :  $\forall w. \langle (\lambda x::w. True), R, V \rangle, w \models^d \varphi \longleftrightarrow w \models^m \Downarrow \varphi$  **apply** *induct by auto*

— Proving faithfulness maximal shallow and minimal shallow

**theorem**  $Faithful3$ :  $\forall w. \langle (\lambda x::w. True), R, V \rangle, w \models^s \Downarrow \varphi \longleftrightarrow w \models^m \Downarrow \varphi$  **apply** *induct by auto*

— Additional check for soundness for the minimal shallow embedding  
**lemma** *Sound1*:  $\models^m \psi \longleftrightarrow (\exists \varphi. \psi = \llbracket \varphi \rrbracket \wedge \models^d \varphi)$  **by** (*smt Faithful2 DefM DefD RelativeTruthD.simps ext[of  $\psi \llbracket x \supset^d x \rrbracket$ ]*)  
**lemma** *Sound2*:  $\models^m \psi \longleftrightarrow (\exists \varphi. \psi = \llbracket \varphi \rrbracket \wedge \models^m \llbracket \varphi \rrbracket)$  **using** *Sound1* **by** *blast*  
**end**

## 7 Appendix: proof automation tests

### 7.1 Tests with the deep embedding

**theory** *PMLinHOL-deep-tests*  
**imports** *PMLinHOL-deep*  
**begin**

— Hilbert calculus: proving that the schematic axioms and rules implied by the embedding

**lemma** *H1*:  $\models^d \varphi \supset^d (\psi \supset^d \varphi)$  **by** *auto*  
**lemma** *H2*:  $\models^d (\varphi \supset^d (\psi \supset^d \gamma)) \supset^d ((\varphi \supset^d \psi) \supset^d (\varphi \supset^d \gamma))$  **by** *auto*  
**lemma** *H3*:  $\models^d (\neg^d \varphi \supset^d \neg^d \psi) \supset^d (\psi \supset^d \varphi)$  **by** *auto*  
**lemma** *MP*:  $\models^d \varphi \implies \models^d (\varphi \supset^d \psi) \implies \models^d \psi$  **by** *auto*

— Reasoning with the Hilbert calculus: interactive and fully automated

**lemma** *HCderived1*:  $\models^d (\varphi \supset^d \varphi)$  — *sledgehammer(HC1 HC2 HC3 MP)* returns:  
**by** (*metis HC1 HC2 MP*)

**proof** —

**have** *1*:  $\models^d \varphi \supset^d ((\psi \supset^d \varphi) \supset^d \varphi)$  **using** *H1* **by** *auto*  
**have** *2*:  $\models^d (\varphi \supset^d ((\psi \supset^d \varphi) \supset^d \varphi)) \supset^d ((\varphi \supset^d (\psi \supset^d \varphi)) \supset^d (\varphi \supset^d \varphi))$  **using**  
*H2* **by** *auto*  
**have** *3*:  $\models^d (\varphi \supset^d (\psi \supset^d \varphi)) \supset^d (\varphi \supset^d \varphi)$  **using** *1 2 MP* **by** *meson*  
**have** *4*:  $\models^d \varphi \supset^d (\psi \supset^d \varphi)$  **using** *H1* **by** *auto*  
**thus** *?thesis* **using** *3 4 MP* **by** *meson*  
**qed**

**lemma** *HCderived2*:  $\models^d \varphi \supset^d (\neg^d \varphi \supset^d \psi)$  **by** (*metis H1 H2 H3 MP*)

**lemma** *HCderived3*:  $\models^d (\neg^d \varphi \supset^d \varphi) \supset^d \varphi$  **by** (*metis H1 H2 H3 MP*)

**lemma** *HCderived4*:  $\models^d (\varphi \supset^d \psi) \supset^d (\neg^d \psi \supset^d \neg^d \varphi)$  **by** *auto*

— Modal logic: the schematic necessitation rule and distribution axiom are implied

**lemma** *Nec*:  $\models^d \varphi \implies \models^d \Box^d \varphi$  **by** *auto*

**lemma** *Dist*:  $\models^d \Box^d (\varphi \supset^d \psi) \supset^d (\Box^d \varphi \supset^d \Box^d \psi)$  **by** *auto*

— Correspondence theory: correct statements

**lemma** *cM*: *reflexive*  $R \longleftrightarrow (\forall \varphi W V. \forall w: W. \langle W, R, V \rangle, w \models^d \Box^d \varphi \supset^d \varphi)$  —  
*sledgehammer*: Proof found **oops**

**lemma** *cBa*: *symmetric*  $R \longrightarrow (\forall \varphi W V. \forall w: W. \langle W, R, V \rangle, w \models^d \varphi \supset^d \Box^d (\Diamond^d \varphi))$   
**by** *auto*

**lemma** *cBb*: *symmetric*  $R \longleftarrow (\forall \varphi W V. \forall w: W. \langle W, R, V \rangle, w \models^d \varphi \supset^d \Box^d (\Diamond^d \varphi))$   
— *sledgehammer*: No proof **oops**

**lemma** *c4a*: *transitive R*  $\longrightarrow$   $(\forall \varphi W V. \forall w: W. \langle W, R, V \rangle, w \models^d \Box^d \varphi \supset^d \Box^d(\Box^d \varphi))$   
**by** (*metis RelativeTruthD.simps*)

**lemma** *c4b*: *transitive R*  $\longleftarrow$   $(\forall \varphi W V. \forall w: W. \langle W, R, V \rangle, w \models^d \Box^d \varphi \supset^d \Box^d(\Box^d \varphi))$   
— sledgehammer: No proof **oops**

— Correspondence theory: incorrect statements

**lemma** *reflexive R*  $\longrightarrow$   $(\forall \varphi W V. \forall w: W. \langle W, R, V \rangle, w \models^d \Box^d \varphi \supset^d \Box^d(\Box^d \varphi))$   
**nitpick**[*card w=3*] **oops** — nitpick: Cex.

— Simple, incorrect validity statements

**lemma**  $\models^d \varphi \supset^d \Box^d \varphi$  **nitpick**[*card w=2, card S= 1*] **oops** — nitpick: Counterexample: modal collapse not implied

**lemma**  $\models^d \Box^d(\Box^d \varphi \supset^d \Box^d \psi) \vee^d \Box^d(\Box^d \psi \supset^d \Box^d \varphi)$  **nitpick**[*card w=3*] **oops**  
— nitpick: Counterexample

**lemma**  $\models^d (\Box^d(\Box^d \varphi)) \supset^d \Box^d(\Box^d \varphi)$  **nitpick**[*card w=2*] **oops** — nitpick: Counterexample

— Implied axiom schemata in S5

**lemma** *KB*: *symmetric R*  $\longrightarrow$   $(\forall \varphi \psi W V. \forall w: W. \langle W, R, V \rangle, w \models^d (\Box^d(\Box^d \varphi)) \supset^d \Box^d(\Box^d \varphi))$  **by** *auto*

**lemma** *K4B*: *symmetric R*  $\wedge$  *transitive R*  $\longrightarrow$   $(\forall \varphi \psi W V. \forall w: W. \langle W, R, V \rangle, w \models^d \Box^d(\Box^d \varphi \supset^d \Box^d \psi) \vee^d \Box^d(\Box^d \psi \supset^d \Box^d \varphi))$  **by** (*smt OrD-def RelativeTruthD.simps*)  
**end**

**theory** *PMLinHOL-deep-further-tests*

**imports** *PMLinHOL-deep-tests*

**begin**

— Implied modal principle

**lemma** *K-Dia*:  $\models^d (\Box^d(\varphi \supset^d \psi)) \supset^d ((\Box^d \varphi) \supset^d \Box^d \psi)$  **by** *auto*

— Example 6.10 of Sider (2009) Logic for Philosophy

**lemma** *T1a*:  $\models^d \Box^d p^d \supset^d ((\Box^d q^d) \supset^d \Box^d(p^d \wedge^d q^d))$  **by** *auto* — fast automation in meta-logic HOL

**lemma** *T1b*:  $\models^d \Box^d p^d \supset^d ((\Box^d q^d) \supset^d \Box^d(p^d \wedge^d q^d))$  — alternative interactive proof in modal object logic K

**proof** —

**have** 1:  $\models^d p^d \supset^d (q^d \supset^d (p^d \wedge^d q^d))$  **unfolding** *AndD-def* **using** *H1 H2 H3 MP* **by** *metis*

**have** 2:  $\models^d \Box^d(p^d \supset^d (q^d \supset^d (p^d \wedge^d q^d)))$  **using** 1 *Nec* **by** *metis*

**have** 3:  $\models^d \Box^d p^d \supset^d \Box^d(q^d \supset^d (p^d \wedge^d q^d))$  **using** 2 *Dist MP* **by** *metis*

**have** 4:  $\models^d (\Box^d(q^d \supset^d (p^d \wedge^d q^d))) \supset^d ((\Box^d q^d) \supset^d \Box^d(p^d \wedge^d q^d))$  **using**

*K-Dia* **by** *metis*

**have** 5:  $\models^d \Box^d p^d \supset^d ((\Box^d q^d) \supset^d \Box^d(p^d \wedge^d q^d))$  **using** 3 4 *H1 H2 MP* **by** *metis*

**thus** *?thesis* .

**qed**

**end**

```

theory PMLinHOL-deep-Loeb-tests
  imports PMLinHOL-deep
begin

```

— Löb axiom: with the deep embedding automated reasoning tools are not very responsive

```

lemma Loeb1:  $\forall \varphi. \models^d \Box^d(\Box^d \varphi \supset^d \varphi) \supset^d \Box^d \varphi$  nitpick[card w=1, card S=1] oops

```

— nitpick: Counterexample

```

lemma Loeb2:  $(\text{conversewf } R \wedge \text{transitive } R) \longrightarrow (\forall \varphi W V. \forall w: W. \langle W, R, V \rangle, w \models^d \Box^d(\Box^d \varphi \supset^d \varphi) \supset^d \Box^d \varphi)$  — sledgehammer: No Proof oops

```

```

lemma Loeb3:  $(\text{conversewf } R \wedge \text{transitive } R) \longleftarrow (\forall \varphi W V. \forall w: W. \langle W, R, V \rangle, w \models^d \Box^d(\Box^d \varphi \supset^d \varphi) \supset^d \Box^d \varphi)$  — sledgehammer: No Proof oops

```

```

lemma Loeb3a:  $\text{conversewf } R \longleftarrow (\forall \varphi W V. \forall w: W. \langle W, R, V \rangle, w \models^d \Box^d(\Box^d \varphi \supset^d \varphi) \supset^d \Box^d \varphi)$  — sledgehammer: No Proof oops

```

```

lemma Loeb3b:  $\text{transitive } R \longleftarrow (\forall \varphi W V. \forall w: W. \langle W, R, V \rangle, w \models^d \Box^d(\Box^d \varphi \supset^d \varphi) \supset^d \Box^d \varphi)$  — sledgehammer: No Proof oops

```

```

lemma Loeb3c:  $\text{irreflexive } R \longleftarrow (\forall \varphi W V. \forall w: W. \langle W, R, V \rangle, w \models^d \Box^d(\Box^d \varphi \supset^d \varphi) \supset^d \Box^d \varphi)$  — sledgehammer: No Proof oops

```

```

end

```

## 7.2 Tests with the maximal shallow embedding

```

theory PMLinHOL-shallow-tests
  imports PMLinHOL-shallow
begin

```

— Hilbert calculus: proving that the schematic axioms and rules implied by the embedding

```

lemma H1:  $\models^s \varphi \supset^s (\psi \supset^s \varphi)$  by auto

```

```

lemma H2:  $\models^s (\varphi \supset^s (\psi \supset^s \gamma)) \supset^s ((\varphi \supset^s \psi) \supset^s (\varphi \supset^s \gamma))$  by auto

```

```

lemma H3:  $\models^s (\neg^s \varphi \supset^s \neg^s \psi) \supset^s (\psi \supset^s \varphi)$  by auto

```

```

lemma MP:  $\models^s \varphi \implies \models^s (\varphi \supset^s \psi) \implies \models^s \psi$  by auto

```

— Reasoning with the Hilbert calculus: interactive and fully automated

```

lemma HCderived1:  $\models^s (\varphi \supset^s \varphi)$  — sledgehammer(HC1 HC2 HC3 MP) returns: by (metis HC1 HC2 MP)

```

```

proof —

```

```

  have 1:  $\models^s \varphi \supset^s ((\psi \supset^s \varphi) \supset^s \varphi)$  using H1 by auto

```

```

  have 2:  $\models^s (\varphi \supset^s ((\psi \supset^s \varphi) \supset^s \varphi)) \supset^s ((\varphi \supset^s (\psi \supset^s \varphi)) \supset^s (\varphi \supset^s \varphi))$  using H2 by auto

```

```

  have 3:  $\models^s (\varphi \supset^s (\psi \supset^s \varphi)) \supset^s (\varphi \supset^s \varphi)$  using 1 2 MP by meson

```

```

  have 4:  $\models^s \varphi \supset^s (\psi \supset^s \varphi)$  using H1 by auto

```

```

  thus ?thesis using 3 4 MP by meson

```

```

qed

```

```

lemma HCderived2:  $\models^s \varphi \supset^s (\neg^s \varphi \supset^s \psi)$  by (metis H1 H2 H3 MP)

```

```

lemma HCderived3:  $\models^s (\neg^s \varphi \supset^s \varphi) \supset^s \varphi$  by (metis H1 H2 H3 MP)

```

```

lemma HCderived4:  $\models^s (\varphi \supset^s \psi) \supset^s (\neg^s \psi \supset^s \neg^s \varphi)$  by auto

```

— Modal logic: the schematic necessitation rule and distribution axiom are implied

**lemma** *Nec*:  $\models^s \varphi \implies \models^s \Box^s \varphi$  **by** *auto*

**lemma** *Dist*:  $\models^s \Box^s(\varphi \supset^s \psi) \supset^s (\Box^s \varphi \supset^s \Box^s \psi)$  **by** *auto*

— Correspondence theory: correct statements

**lemma** *cM:reflexive*  $R \iff (\forall \varphi W V. \forall w:W. \langle W,R,V \rangle, w \models^s \Box^s \varphi \supset^s \varphi)$  —  
sledgehammer: Proof found **oops**

**lemma** *cBa*: *symmetric*  $R \implies (\forall \varphi W V. \forall w:W. \langle W,R,V \rangle, w \models^s \varphi \supset^s \Box^s(\Diamond^s \varphi))$   
**by** *auto*

**lemma** *cBb*: *symmetric*  $R \leftarrow (\forall \varphi W V. \forall w:W. \langle W,R,V \rangle, w \models^s \varphi \supset^s \Box^s(\Diamond^s \varphi))$   
— sledgehammer: No proof **oops**

**lemma** *cAa*: *transitive*  $R \implies (\forall \varphi W V. \forall w:W. \langle W,R,V \rangle, w \models^s \Box^s \varphi \supset^s \Box^s(\Box^s \varphi))$   
**by** (*smt DefS*)

**lemma** *cAb*: *transitive*  $R \leftarrow (\forall \varphi W V. \forall w:W. \langle W,R,V \rangle, w \models^s \Box^s \varphi \supset^s \Box^s(\Box^s \varphi))$   
— sledgehammer: No proof **oops**

— Correspondence theory: incorrect statements

**lemma** *reflexive*  $R \implies (\forall \varphi W V. \forall w:W. \langle W,R,V \rangle, w \models^s \Box^s \varphi \supset^s \Box^s(\Box^s \varphi))$   
**nitpick**[*card w=3*] **oops** — nitpick: Counterexample

— Simple, incorrect validity statements

**lemma**  $\models^s \varphi \supset^s \Box^s \varphi$  **nitpick**[*card w=2, card S= 1*] **oops** — nitpick: Counterexample: modal collapse not implied

**lemma**  $\models^s \Box^s(\Box^s \varphi \supset^s \Box^s \psi) \vee^s \Box^s(\Box^s \psi \supset^s \Box^s \varphi)$  **oops** — nitpick[*card w=3*]  
returns: unknown

**lemma**  $\models^s (\Diamond^s(\Box^s \varphi)) \supset^s \Box^s(\Diamond^s \varphi)$  **nitpick**[*card w=2*] **oops** — nitpick: Counterexample

— Implied axiom schemata in S5

**lemma** *KB*: *symmetric*  $R \implies (\forall \varphi \psi W V. \forall w:W. \langle W,R,V \rangle, w \models^s (\Diamond^s(\Box^s \varphi)) \supset^s \Box^s(\Diamond^s \varphi))$  **by** *auto*

**lemma** *K4B*: *symmetric*  $R \wedge$  *transitive*  $R \implies (\forall \varphi \psi W V. \forall w:W. \langle W,R,V \rangle, w \models^s \Box^s(\Box^s \varphi \supset^s \Box^s \psi) \vee^s \Box^s(\Box^s \psi \supset^s \Box^s \varphi))$  **by** (*smt DefS*)  
**end**

**theory** *PMLinHOL-shallow-further-tests*

**imports** *PMLinHOL-shallow-tests*

**begin**

— Implied modal principle

**lemma** *K-Dia*:  $\models^s (\Box^s(\varphi \supset^s \psi)) \supset^s ((\Diamond^s \varphi) \supset^s \Diamond^s \psi)$  **by** *auto*

— Example 6.10 of Sider (2009) Logic for Philosophy

**lemma** *T1a*:  $\models^s \Box^s p^s \supset^s ((\Diamond^s q^s) \supset^s \Diamond^s (p^s \wedge^s q^s))$  **by** *auto* — fast automation in meta-logic HOL

**lemma** *T1b*:  $\models^s \Box^s p^s \supset^s ((\Diamond^s q^s) \supset^s \Diamond^s (p^s \wedge^s q^s))$  — alternative interactive proof in modal object logic K

**proof** —

**have** 1:  $\models^s p^s \supset^s (q^s \supset^s (p^s \wedge^s q^s))$  **unfolding** *AndS-def* **using** *H1 H2 H3*

*MP by metis*  
**have 2:**  $\models^s \Box^s (p^s \supset^s (q^s \supset^s (p^s \wedge^s q^s)))$  **using 1 Nec by metis**  
**have 3:**  $\models^s \Box^s p^s \supset^s \Box^s (q^s \supset^s (p^s \wedge^s q^s))$  **using 2 Dist MP by metis**  
**have 4:**  $\models^s (\Box^s (q^s \supset^s (p^s \wedge^s q^s))) \supset^s ((\Diamond^s q^s) \supset^s \Diamond^s (p^s \wedge^s q^s))$  **using K-Dia**  
**by metis**  
**have 5:**  $\models^s \Box^s p^s \supset^s ((\Diamond^s q^s) \supset^s \Diamond^s (p^s \wedge^s q^s))$  **using 3 4 H1 H2 MP by metis**  
  
**thus ?thesis .**  
**qed**  
**end**

**theory PMLinHOL-shallow-Loeb-tests**

**imports PMLinHOL-shallow**

**begin**

— Löb axiom: with the minimal shallow embedding automated reasoning tools are still partly responsive

**lemma Loeb1:**  $\forall \varphi. \models^s \Box^s (\Box^s \varphi \supset^s \varphi) \supset^s \Box^s \varphi$  **nitpick[card w=1, card S=1] oops**

— nitpick: Counterexample

**lemma Loeb2:**  $(\text{conversewf } R \wedge \text{transitive } R) \longrightarrow (\forall \varphi W V. \forall w: W. \langle W, R, V \rangle, w \models^s \Box^s (\Box^s \varphi \supset^s \varphi) \supset^s \Box^s \varphi)$  — sledgehammer: Proof found **oops**

**lemma Loeb3:**  $(\text{conversewf } R \wedge \text{transitive } R) \longleftarrow (\forall \varphi W V. \forall w: W. \langle W, R, V \rangle, w \models^s \Box^s (\Box^s \varphi \supset^s \varphi) \supset^s \Box^s \varphi)$  — sledgehammer: No Proof **oops**

**lemma Loeb3a:**  $\text{conversewf } R \longleftarrow (\forall \varphi W V. \forall w: W. \langle W, R, V \rangle, w \models^s \Box^s (\Box^s \varphi \supset^s \varphi) \supset^s \Box^s \varphi)$  — sledgehammer: Proof found **oops**

**lemma Loeb3b:**  $\text{transitive } R \longleftarrow (\forall \varphi W V. \forall w: W. \langle W, R, V \rangle, w \models^s \Box^s (\Box^s \varphi \supset^s \varphi) \supset^s \Box^s \varphi)$  — sledgehammer: No Proof **oops**

**lemma Loeb3c:**  $\text{irreflexive } R \longleftarrow (\forall \varphi W V. \forall w: W. \langle W, R, V \rangle, w \models^s \Box^s (\Box^s \varphi \supset^s \varphi) \supset^s \Box^s \varphi)$  — sledgehammer: Proof found **oops**

**end**

### 7.3 Tests with the minimal shallow embedding

**theory PMLinHOL-shallow-minimal-tests**

**imports PMLinHOL-shallow-minimal**

**begin**

— Hilbert calculus: proving that the schematic axioms and rules implied by the embedding

**lemma H1:**  $\models^m \varphi \supset^m (\psi \supset^m \varphi)$  **by auto**

**lemma H2:**  $\models^m (\varphi \supset^m (\psi \supset^m \gamma)) \supset^m ((\varphi \supset^m \psi) \supset^m (\varphi \supset^m \gamma))$  **by auto**

**lemma H3:**  $\models^m (\neg^m \varphi \supset^m \neg^m \psi) \supset^m (\psi \supset^m \varphi)$  **by auto**

**lemma MP:**  $\models^m \varphi \implies \models^m (\varphi \supset^m \psi) \implies \models^m \psi$  **by auto**

— Reasoning with the Hilbert calculus: interactive and fully automated

**lemma HCderived1:**  $\models^m (\varphi \supset^m \varphi)$  — sledgehammer(HC1 HC2 HC3 MP) returns: by (metis HC1 HC2 MP)

**proof** —

**have 1:**  $\models^m \varphi \supset^m ((\psi \supset^m \varphi) \supset^m \varphi)$  **using H1 by auto**

**have 2:**  $\models^m (\varphi \supset^m ((\psi \supset^m \varphi) \supset^m \varphi)) \supset^m ((\varphi \supset^m (\psi \supset^m \varphi)) \supset^m (\varphi \supset^m \varphi))$

using *H2* by *auto*

have 3:  $\models^m (\varphi \supset^m (\psi \supset^m \varphi)) \supset^m (\varphi \supset^m \varphi)$  using 1 2 *MP* by *meson*

have 4:  $\models^m \varphi \supset^m (\psi \supset^m \varphi)$  using *H1* by *auto*

thus ?thesis using 3 4 *MP* by *meson*

qed

lemma *HCderived2*:  $\models^m \varphi \supset^m (\neg^m \varphi \supset^m \psi)$  by (*metis H1 H2 H3 MP*)

lemma *HCderived3*:  $\models^m (\neg^m \varphi \supset^m \varphi) \supset^m \varphi$  by (*metis H1 H2 H3 MP*)

lemma *HCderived4*:  $\models^m (\varphi \supset^m \psi) \supset^m (\neg^m \psi \supset^m \neg^m \varphi)$  by *auto*

— Modal logic: the schematic necessitation rule and distribution axiom are implied

lemma *Nec*:  $\models^m \varphi \implies \models^m \Box^m \varphi$  by (*smt DefM*)

lemma *Dist*:  $\models^m \Box^m (\varphi \supset^m \psi) \supset^m (\Box^m \varphi \supset^m \Box^m \psi)$  by *auto*

— Correspondence theory: correct statements

lemma *cM:reflexive R*:  $\longleftrightarrow (\forall \varphi. \models^m \Box^m \varphi \supset^m \varphi)$  by *auto*

lemma *cBa:symmetric R*:  $\longrightarrow (\forall \varphi. \models^m \varphi \supset^m \Box^m \Diamond^m \varphi)$  by *auto*

lemma *cBb:symmetric R*:  $\longleftarrow (\forall \varphi. \models^m \varphi \supset^m \Box^m \Diamond^m \varphi)$  by (*metis DefM*)

lemma *c4a:transitive R*:  $\longrightarrow (\forall \varphi. \models^m \Box^m \varphi \supset^m \Box^m (\Box^m \varphi))$  by (*smt DefM*)

lemma *c4b:transitive R*:  $\longleftarrow (\forall \varphi. \models^m \Box^m \varphi \supset^m \Box^m (\Box^m \varphi))$  by *auto*

— Correspondence theory: incorrect statements

lemma *reflexive R*:  $\longrightarrow (\forall \varphi. \models^m \Box^m \varphi \supset^m \Box^m (\Box^m \varphi))$  **nitpick**[*card w=3,show-all*]

**oops** — nitpick: Counterexample

— Simple, incorrect validity statements

lemma  $\models^m \varphi \supset^m \Box^m \varphi$  **nitpick**[*card w=2, card S= 1*] **oops** — nitpick: Counterexample: modal collapse not implied

lemma  $\models^m \Box^m (\Box^m \varphi \supset^m \Box^m \psi) \vee^m \Box^m (\Box^m \psi \supset^m \Box^m \varphi)$  **nitpick**[*card w=3*] **oops**

— nitpick: Counterexample

lemma  $\models^m (\Diamond^m (\Box^m \varphi)) \supset^m \Box^m (\Diamond^m \varphi)$  **nitpick**[*card w=2*] **oops** — nitpick: Counterexample

— Implied axiom schemata in S5

lemma *KB:symmetric R*:  $\longrightarrow (\forall \varphi \psi. \models^m (\Diamond^m (\Box^m \varphi)) \supset^m \Box^m (\Diamond^m \varphi))$  by *auto*

lemma *K4B:symmetric R  $\wedge$  transitive R*:  $\longrightarrow (\forall \varphi \psi. \models^m \Box^m (\Box^m \varphi \supset^m \Box^m \psi) \vee^m \Box^m (\Box^m \psi \supset^m \Box^m \varphi))$  by (*smt DefM*)

end

theory *PMLinHOL-shallow-minimal-further-tests*

imports *PMLinHOL-shallow-minimal-tests*

begin

— Implied modal principle

lemma *K-Dia*:  $\models^m (\Box^m (\varphi \supset^m \psi)) \supset^m ((\Diamond^m \varphi) \supset^m \Diamond^m \psi)$  by *auto*

— Example 6.10 of Sider (2009) Logic for Philosophy

lemma *T1a*:  $\models^m \Box^m p^m \supset^m ((\Diamond^m q^m) \supset^m \Diamond^m (p^m \wedge^m q^m))$  by *auto* — fast automation in meta-logic HOL

**lemma** *T1b*:  $\models^m \Box^m p^m \supset^m ((\Diamond^m q^m) \supset^m \Diamond^m (p^m \wedge^m q^m))$  — alternative interactive proof in modal object logic K

**proof** –

**have** 1:  $\models^m p^m \supset^m (q^m \supset^m (p^m \wedge^m q^m))$  **unfolding** *AndM-def* **using** *H1 H2 H3 MP* **by** *metis*

**have** 2:  $\models^m \Box^m (p^m \supset^m (q^m \supset^m (p^m \wedge^m q^m)))$  **using** 1 *Nec* **by** *metis*

**have** 3:  $\models^m \Box^m p^m \supset^m \Box^m (q^m \supset^m (p^m \wedge^m q^m))$  **using** 2 *Dist MP* **by** *metis*

**have** 4:  $\models^m (\Box^m (q^m \supset^m (p^m \wedge^m q^m))) \supset^m ((\Diamond^m q^m) \supset^m \Diamond^m (p^m \wedge^m q^m))$

**using** *K-Dia* **by** *metis*

**have** 5:  $\models^m \Box^m p^m \supset^m ((\Diamond^m q^m) \supset^m \Diamond^m (p^m \wedge^m q^m))$  **using** 3 4 *H1 H2 MP* **by** *metis*

**thus** *?thesis* .

**qed**

**end**

**theory** *PMLinHOL-shallow-minimal-Loeb-tests*

**imports** *PMLinHOL-shallow-minimal*

**begin**

— Löb axiom: with the minimal shallow embedding automated reasoning tools are still partly responsive

**lemma** *Loeb1*:  $\forall \varphi. \models^m \Box^m (\Box^m \varphi \supset^m \varphi) \supset^m \Box^m \varphi$  **nitpick**[*card w=1, card S=1*]

**oops** — nitpick: Counterexample.

**lemma** *Loeb2*:  $(\text{conversew } R \wedge \text{transitive } R) \longrightarrow (\forall \varphi. \models^m \Box^m (\Box^m \varphi \supset^m \varphi) \supset^m \Box^m \varphi)$  — sh: Proof found **oops**

**lemma** *Loeb3*:  $(\text{conversew } R \wedge \text{transitive } R) \longleftarrow (\forall \varphi. \models^m \Box^m (\Box^m \varphi \supset^m \varphi) \supset^m \Box^m \varphi)$  — sh: No Proof **oops**

**lemma** *Loeb3a*:  $\text{conversew } R \longleftarrow (\forall \varphi. \models^m \Box^m (\Box^m \varphi \supset^m \varphi) \supset^m \Box^m \varphi)$  **unfolding** *DefM* **by** *blast*

**lemma** *Loeb3b*:  $\text{transitive } R \longleftarrow (\forall \varphi. \models^m \Box^m (\Box^m \varphi \supset^m \varphi) \supset^m \Box^m \varphi)$  — sledgehammer: No Proof **oops**

**lemma** *Loeb3c*:  $\text{irreflexive } R \longleftarrow (\forall \varphi. \models^m \Box^m (\Box^m \varphi \supset^m \varphi) \supset^m \Box^m \varphi)$  — sledgehammer: Proof found **oops**

**end**

## 8 Test Examples: Formula classification

### 8.1 Tests with the deep embedding: axiom schemata

**theory** *PMLinHOL-deep-further-tests-1*

**imports** *PMLinHOL-deep-tests*

**begin**

— What is the weakest modal logic in which the following test formulas F1-F10 are provable?

— Test with schematic axioms

**consts**  $\varphi::PML \ \psi::PML$

**abbreviation**(*input*)  $F1 \equiv (\Diamond^d (\Diamond^d \varphi)) \supset^d \Diamond^d \varphi$  — holds in K4

**abbreviation**(*input*)  $F2 \equiv (\Diamond^d (\Box^d \varphi)) \supset^d \Box^d (\Diamond^d \varphi)$  — holds in KB

**abbreviation**(*input*)  $F3 \equiv (\Diamond^d (\Box^d \varphi)) \supset^d \Box^d \varphi$  — holds in KB4

**abbreviation**(*input*)  $F4 \equiv (\Box^d(\Diamond^d(\Box^d(\Diamond^d\varphi)))) \supset^d \Box^d(\Diamond^d\varphi)$  — holds in KB/K4  
**abbreviation**(*input*)  $F5 \equiv (\Diamond^d(\varphi \wedge^d (\Diamond^d\psi))) \supset^d ((\Diamond^d\varphi) \wedge^d (\Diamond^d\psi))$  — holds in K4  
**abbreviation**(*input*)  $F6 \equiv ((\Box^d(\varphi \supset^d \psi)) \wedge^d (\Diamond^d(\Box^d(\neg^d\psi)))) \supset^d \neg^d(\Diamond^d\psi)$  — holds in KB4  
**abbreviation**(*input*)  $F7 \equiv (\Diamond^d\varphi) \supset^d (\Box^d(\varphi \vee^d \Diamond^d\varphi))$  — holds in KB4  
**abbreviation**(*input*)  $F8 \equiv (\Diamond^d(\Box^d\varphi)) \supset^d (\varphi \vee^d \Diamond^d\varphi)$  — holds in KT/KB  
**abbreviation**(*input*)  $F9 \equiv ((\Box^d(\Diamond^d\varphi)) \wedge^d (\Box^d(\Diamond^d(\neg^d\varphi)))) \supset^d \Diamond^d(\Diamond^d\varphi)$  — holds in KT  
**abbreviation**(*input*)  $F10 \equiv ((\Box^d(\varphi \supset^d \Box^d\psi)) \wedge^d (\Box^d(\Diamond^d(\neg^d\psi)))) \supset^d \neg^d(\Box^d\psi)$  — holds in KT

**declare** *imp-cong*[*cong del*]

**experiment begin**

**lemma**  $S5: \forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d\varphi) \supset^d \varphi) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V \rangle, w \models^d F1$

— nitpick[expect=none] — sledgehammer — none — proof

**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof

**by** (*smt* (*z3*) *PML.simps*(22))

**lemma**  $S4: \forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d\varphi) \supset^d \varphi) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V \rangle, w \models^d F1$

— nitpick[expect=none] — sledgehammer — none — proof

**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf

**by** (*smt* (*z3*) *PML.simps*(22))

**lemma**  $KB4: \forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V \rangle, w \models^d F1$

— nitpick[expect=none] — sledgehammer — none — proof

**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof

**by** (*smt* (*z3*) *PML.simps*(22))

**lemma**  $KTB: \forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d\varphi) \supset^d \varphi) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \longrightarrow \langle W,R,V \rangle, w \models^d F1$

— nitpick[expect=none] — sledgehammer — none — no prf

**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf

**oops**

**lemma**  $KT: \forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d\varphi) \supset^d \varphi) \longrightarrow \langle W,R,V \rangle, w \models^d F1$

— nitpick[expect=none] — sledgehammer — none — no prf

**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf

**oops**

**lemma**  $KB: \forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \longrightarrow \langle W,R,V \rangle, w \models^d F1$

— nitpick[expect=none] — sledgehammer — none — no prf

**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf

**oops**

**lemma**  $K4: \forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V \rangle, w \models^d F1$

— nitpick[expect=none] — sledgehammer — none — proof

**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof

```

  by (smt (z3) PML.simps(22))
lemma K:  $\forall w:W. \langle W,R,V \rangle, w \models^d F1$ 
  — nitpick[expect=genuine] — sledgehammer — ctex — no prf
  apply simp — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
end

experiment begin
lemma S5:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi) \supset^d \varphi) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^d \varphi \supset^d \Box^d(\Diamond^d \varphi)) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi) \supset^d \Box^d(\Box^d \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^d F2$ 
  — nitpick[expect=none] — sledgehammer — none — no prf
  apply simp — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma S4:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi) \supset^d \varphi) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi) \supset^d \Box^d(\Box^d \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^d F2$ 
  — nitpick[expect=none] — sledgehammer — none — no prf
  apply simp — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma KB4:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d \varphi \supset^d \Box^d(\Diamond^d \varphi)) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi) \supset^d \Box^d(\Box^d \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^d F2$ 
  — nitpick[expect=none] — sledgehammer — none — no prf
  apply simp — nitpick[expect=unknown] — sledgehammer — unkn — proof
  by (smt (z3) PML.simps(22))
lemma KTB:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi) \supset^d \varphi) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^d \varphi \supset^d \Box^d(\Diamond^d \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^d F2$ 
  — nitpick[expect=none] — sledgehammer — none — no prf
  apply simp — nitpick[expect=unknown] — sledgehammer — unkn — proof
  by (smt (z3) PML.simps(22))
lemma KT:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi) \supset^d \varphi) \longrightarrow \langle W,R,V \rangle, w \models^d F2$ 
  — nitpick[expect=none] — sledgehammer — none — no prf
  apply simp — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma KB:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d \varphi \supset^d \Box^d(\Diamond^d \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^d F2$ 
  — nitpick[expect=none] — sledgehammer — none — proof
  apply simp — nitpick[expect=unknown] — sledgehammer — unkn — proof
  by (smt (z3) PML.simps(22))
lemma K4:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi) \supset^d \Box^d(\Box^d \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^d F2$ 
  — nitpick[expect=none] — sledgehammer — none — no prf
  apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma K:  $\forall w:W. \langle W,R,V \rangle, w \models^d F2$ 
  — nitpick[expect=genuine] — sledgehammer — ctex — no prf
  apply simp — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
end

```

**experiment begin**

**lemma** *S5*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi) \supset^d \varphi) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^d \varphi \supset^d \Box^d(\Diamond^d \varphi)) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi) \supset^d \Box^d(\Box^d \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^d F3$

— nitpick[expect=none] — sledgehammer — none — no prf

**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf

**oops**

**lemma** *S4*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi) \supset^d \varphi) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi) \supset^d \Box^d(\Box^d \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^d F3$

— nitpick[expect=none] — sledgehammer — none — no prf

**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf

**oops**

**lemma** *KB4*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d \varphi \supset^d \Box^d(\Diamond^d \varphi)) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi) \supset^d \Box^d(\Box^d \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^d F3$

— nitpick[expect=none] — sledgehammer — none — no prf

**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf

**oops**

**lemma** *KTB*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi) \supset^d \varphi) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^d \varphi \supset^d \Box^d(\Diamond^d \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^d F3$

— nitpick[expect=none] — sledgehammer — none — no prf

**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf

**oops**

**lemma** *KT*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi) \supset^d \varphi) \longrightarrow \langle W,R,V \rangle, w \models^d F3$

— nitpick[expect=none] — sledgehammer — none — no prf

**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf

**oops**

**lemma** *KB*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d \varphi \supset^d \Box^d(\Diamond^d \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^d F3$

— nitpick[expect=none] — sledgehammer — none — no prf

**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf

**oops**

**lemma** *K4*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi) \supset^d \Box^d(\Box^d \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^d F3$

— nitpick[expect=none] — sledgehammer — none — no prf

**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf

**oops**

**lemma** *K*:  $\forall w:W. \langle W,R,V \rangle, w \models^d F3$

— nitpick[expect=genuine] — sledgehammer — ctext — no prf

**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf

**oops**

**end**

**experiment begin**

**lemma** *S5*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi) \supset^d \varphi) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^d \varphi \supset^d \Box^d(\Diamond^d \varphi)) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi) \supset^d \Box^d(\Box^d \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^d F4$

— nitpick[expect=none] — sledgehammer — none — no prf

**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf

**oops**

**lemma**  $S_4$ :  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi) \supset^d \varphi) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi) \supset^d \Box^d(\Box^d \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^d F_4$   
— nitpick[expect=none] — sledgehammer — none — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**

**lemma**  $KB_4$ :  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d \varphi \supset^d \Box^d(\Diamond^d \varphi)) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi) \supset^d \Box^d(\Box^d \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^d F_4$   
— nitpick[expect=none] — sledgehammer — none — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**

**lemma**  $KTB$ :  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi) \supset^d \varphi) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^d \varphi \supset^d \Box^d(\Diamond^d \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^d F_4$   
— nitpick[expect=none] — sledgehammer — none — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**

**lemma**  $KT$ :  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi) \supset^d \varphi) \longrightarrow \langle W,R,V \rangle, w \models^d F_4$   
— nitpick[expect=none] — sledgehammer — none — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**

**lemma**  $KB$ :  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d \varphi \supset^d \Box^d(\Diamond^d \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^d F_4$   
— nitpick[expect=none] — sledgehammer — none — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**

**lemma**  $K_4$ :  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi) \supset^d \Box^d(\Box^d \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^d F_4$   
— nitpick[expect=none] — sledgehammer — none — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**

**lemma**  $K$ :  $\forall w:W. \langle W,R,V \rangle, w \models^d F_4$   
— nitpick[expect=genuine] — sledgehammer — ctext — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**  
**end**

### experiment begin

**lemma**  $S_5$ :  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi) \supset^d \varphi) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^d \varphi \supset^d \Box^d(\Diamond^d \varphi)) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi) \supset^d \Box^d(\Box^d \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^d F_5$   
— nitpick[expect=none] — sledgehammer — none — proof  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**by** (*smt* (*z3*) *PML.simps*(22))

**lemma**  $S_4$ :  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi) \supset^d \varphi) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi) \supset^d \Box^d(\Box^d \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^d F_5$   
— nitpick[expect=none] — sledgehammer — none — proof  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**by** (*smt* (*z3*) *PML.simps*(22))

**lemma**  $KB_4$ :  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d \varphi \supset^d \Box^d(\Diamond^d \varphi)) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi) \supset^d \Box^d(\Box^d \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^d F_5$

— nitpick[expect=none] — sledgehammer — none — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**by** (*smt* (*z3*) *PML.simps*(22))

**lemma** *KTB*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi) \supset^d \varphi) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^d \varphi \supset^d \Box^d(\Diamond^d \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^d F5$   
 — nitpick[expect=none] — sledgehammer — none — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**

**lemma** *KT*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi) \supset^d \varphi) \longrightarrow \langle W,R,V \rangle, w \models^d F5$   
 — nitpick[expect=none] — sledgehammer — none — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**

**lemma** *KB*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d \varphi \supset^d \Box^d(\Diamond^d \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^d F5$   
 — nitpick[expect=none] — sledgehammer — none — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**

**lemma** *K4*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi) \supset^d \Box^d(\Box^d \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^d F5$   
 — nitpick[expect=none] — sledgehammer — none — proof  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**by** (*smt* (*z3*) *PML.simps*(22))

**lemma** *K*:  $\forall w:W. \langle W,R,V \rangle, w \models^d F5$   
 — nitpick[expect=none] — sledgehammer — none — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**  
**end**

**experiment begin**

**lemma** *S5*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi) \supset^d \varphi) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^d \varphi \supset^d \Box^d(\Diamond^d \varphi)) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi) \supset^d \Box^d(\Box^d \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^d F6$   
 — nitpick[expect=none] — sledgehammer — none — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**

**lemma** *S4*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi) \supset^d \varphi) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi) \supset^d \Box^d(\Box^d \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^d F6$   
 — nitpick[expect=none] — sledgehammer — none — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**

**lemma** *KB4*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d \varphi \supset^d \Box^d(\Diamond^d \varphi)) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi) \supset^d \Box^d(\Box^d \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^d F6$   
 — nitpick[expect=none] — sledgehammer — none — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**

**lemma** *KTB*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi) \supset^d \varphi) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^d \varphi \supset^d \Box^d(\Diamond^d \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^d F6$   
 — nitpick[expect=none] — sledgehammer — none — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf

**oops**  
**lemma** *KT*:  $\forall w: W. (\forall \varphi. \langle W, R, V \rangle, w \models^d (\Box^d \varphi) \supset^d \varphi) \longrightarrow \langle W, R, V \rangle, w \models^d F6$   
— nitpick[expect=none] — sledgehammer — none — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**  
**lemma** *KB*:  $\forall w: W. (\forall \varphi. \langle W, R, V \rangle, w \models^d \varphi \supset^d \Box^d(\Diamond^d \varphi)) \longrightarrow \langle W, R, V \rangle, w \models^d F6$   
— nitpick[expect=none] — sledgehammer — none — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**  
**lemma** *K4*:  $\forall w: W. (\forall \varphi. \langle W, R, V \rangle, w \models^d (\Box^d \varphi) \supset^d \Box^d(\Box^d \varphi)) \longrightarrow \langle W, R, V \rangle, w \models^d F6$   
— nitpick[expect=none] — sledgehammer — none — no prf  
**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctext — no prf  
**oops**  
**lemma** *K*:  $\forall w: W. \langle W, R, V \rangle, w \models^d F6$   
— nitpick[expect=none] — sledgehammer — none — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**  
**end**

**experiment begin**  
**lemma** *S5*:  $\forall w: W. (\forall \varphi. \langle W, R, V \rangle, w \models^d (\Box^d \varphi) \supset^d \varphi) \wedge (\forall \varphi. \langle W, R, V \rangle, w \models^d \varphi \supset^d \Box^d(\Diamond^d \varphi)) \wedge (\forall \varphi. \langle W, R, V \rangle, w \models^d (\Box^d \varphi) \supset^d \Box^d(\Box^d \varphi)) \longrightarrow \langle W, R, V \rangle, w \models^d F7$   
— nitpick[expect=none] — sledgehammer — none — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**  
**lemma** *S4*:  $\forall w: W. (\forall \varphi. \langle W, R, V \rangle, w \models^d (\Box^d \varphi) \supset^d \varphi) \wedge (\forall \varphi. \langle W, R, V \rangle, w \models^d (\Box^d \varphi) \supset^d \Box^d(\Box^d \varphi)) \longrightarrow \langle W, R, V \rangle, w \models^d F7$   
— nitpick[expect=none] — sledgehammer — none — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**  
**lemma** *KB4*:  $\forall w: W. (\forall \varphi. \langle W, R, V \rangle, w \models^d \varphi \supset^d \Box^d(\Diamond^d \varphi)) \wedge (\forall \varphi. \langle W, R, V \rangle, w \models^d (\Box^d \varphi) \supset^d \Box^d(\Box^d \varphi)) \longrightarrow \langle W, R, V \rangle, w \models^d F7$   
— nitpick[expect=none] — sledgehammer — none — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**  
**lemma** *KTB*:  $\forall w: W. (\forall \varphi. \langle W, R, V \rangle, w \models^d (\Box^d \varphi) \supset^d \varphi) \wedge (\forall \varphi. \langle W, R, V \rangle, w \models^d \varphi \supset^d \Box^d(\Diamond^d \varphi)) \longrightarrow \langle W, R, V \rangle, w \models^d F7$   
— nitpick[expect=none] — sledgehammer — none — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**  
**lemma** *KT*:  $\forall w: W. (\forall \varphi. \langle W, R, V \rangle, w \models^d (\Box^d \varphi) \supset^d \varphi) \longrightarrow \langle W, R, V \rangle, w \models^d F7$   
— nitpick[expect=none] — sledgehammer — none — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**  
**lemma** *KB*:  $\forall w: W. (\forall \varphi. \langle W, R, V \rangle, w \models^d \varphi \supset^d \Box^d(\Diamond^d \varphi)) \longrightarrow \langle W, R, V \rangle, w \models^d F7$

— nitpick[expect=none] — sledgehammer — none — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**  
**lemma** *K4*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi) \supset^d \Box^d(\Box^d \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^d F7$   
 — nitpick[expect=none] — sledgehammer — none — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**  
**lemma** *K*:  $\forall w:W. \langle W,R,V \rangle, w \models^d F7$   
 — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**  
**end**

### experiment begin

**lemma** *S5*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi) \supset^d \varphi) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^d \varphi \supset^d \Box^d(\Box^d \varphi)) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi) \supset^d \Box^d(\Box^d \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^d F8$

— nitpick[expect=none] — sledgehammer — none — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**  
**lemma** *S4*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi) \supset^d \varphi) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi) \supset^d \Box^d(\Box^d \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^d F8$   
 — nitpick[expect=none] — sledgehammer — none — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**  
**lemma** *KB4*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d \varphi \supset^d \Box^d(\Box^d \varphi)) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi) \supset^d \Box^d(\Box^d \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^d F8$   
 — nitpick[expect=none] — sledgehammer — none — proof  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**by** (*smt* (*z3*) *PML.simps(22)*)  
**lemma** *KTB*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi) \supset^d \varphi) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^d \varphi \supset^d \Box^d(\Box^d \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^d F8$   
 — nitpick[expect=none] — sledgehammer — none — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**by** (*smt* (*z3*) *PML.simps(22)*)  
**lemma** *KT*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi) \supset^d \varphi) \longrightarrow \langle W,R,V \rangle, w \models^d F8$   
 — nitpick[expect=none] — sledgehammer — none — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**  
**lemma** *KB*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d \varphi \supset^d \Box^d(\Box^d \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^d F8$   
 — nitpick[expect=none] — sledgehammer — none — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**  
**lemma** *K4*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi) \supset^d \Box^d(\Box^d \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^d F8$   
 — nitpick[expect=none] — sledgehammer — none — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf

```

oops
lemma K:  $\forall w:W. \langle W,R,V \rangle, w \models^d F8$ 
  — nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply simp — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
end

experiment begin
lemma S5:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi) \supset^d \varphi) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^d \varphi \supset^d \Box^d(\Diamond^d \varphi)) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi) \supset^d \Box^d(\Box^d \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^d F9$ 
  — nitpick[expect=none] — sledgehammer — none — proof
apply simp — nitpick[expect=unknown] — sledgehammer — unkn — proof
by (metis (mono-tags, lifting) PML.simps(22))
lemma S4:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi) \supset^d \varphi) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi) \supset^d \Box^d(\Box^d \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^d F9$ 
  — nitpick[expect=none] — sledgehammer — none — proof
apply simp — nitpick[expect=unknown] — sledgehammer — unkn — proof
by (metis (mono-tags, lifting) PML.simps(22))
lemma KB4:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d \varphi \supset^d \Box^d(\Diamond^d \varphi)) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi) \supset^d \Box^d(\Box^d \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^d F9$ 
  — nitpick[expect=none] — sledgehammer — none — no prf
apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma KTB:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi) \supset^d \varphi) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^d \varphi \supset^d \Box^d(\Diamond^d \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^d F9$ 
  — nitpick[expect=none] — sledgehammer — none — proof
apply simp — nitpick[expect=unknown] — sledgehammer — unkn — proof
by (metis (mono-tags, lifting) PML.simps(22))
lemma KT:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi) \supset^d \varphi) \longrightarrow \langle W,R,V \rangle, w \models^d F9$ 
  — nitpick[expect=none] — sledgehammer — none — proof
apply simp — nitpick[expect=unknown] — sledgehammer — unkn — proof
by (metis (mono-tags, lifting) PML.simps(22))
lemma KB:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d \varphi \supset^d \Box^d(\Diamond^d \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^d F9$ 
  — nitpick[expect=none] — sledgehammer — none — no prf
apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma K4:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi) \supset^d \Box^d(\Box^d \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^d F9$ 
  — nitpick[expect=none] — sledgehammer — none — no prf
apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma K:  $\forall w:W. \langle W,R,V \rangle, w \models^d F9$ 
  — nitpick[expect=none] — sledgehammer — none — no prf
apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
end

```

**experiment begin**

**lemma** *S5*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi \supset^d \varphi) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^d \varphi \supset^d \Box^d(\Diamond^d \varphi))) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi \supset^d \Box^d(\Box^d \varphi))) \longrightarrow \langle W,R,V \rangle, w \models^d F10$

— nitpick[expect=none] — sledgehammer — none — proof

**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof

**by** (*metis (mono-tags, lifting) PML.simps(22)*)

**lemma** *S4*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi \supset^d \varphi) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi \supset^d \Box^d(\Box^d \varphi))) \longrightarrow \langle W,R,V \rangle, w \models^d F10$

— nitpick[expect=none] — sledgehammer — none — proof

**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof

**by** (*metis (mono-tags, lifting) PML.simps(22)*)

**lemma** *KB4*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d \varphi \supset^d \Box^d(\Diamond^d \varphi)) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi \supset^d \Box^d(\Box^d \varphi))) \longrightarrow \langle W,R,V \rangle, w \models^d F10$

— nitpick[expect=none] — sledgehammer — none — no prf

**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf

**oops**

**lemma** *KTB*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi \supset^d \varphi) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^d \varphi \supset^d \Box^d(\Diamond^d \varphi))) \longrightarrow \langle W,R,V \rangle, w \models^d F10$

— nitpick[expect=none] — sledgehammer — none — proof

**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf

**by** (*smt (verit) PML.simps(22,24)*)

**lemma** *KT*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi \supset^d \varphi) \longrightarrow \langle W,R,V \rangle, w \models^d F10$

— nitpick[expect=none] — sledgehammer — none — proof

**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf

**by** (*smt (verit) PML.simps(22,24)*)

**lemma** *KB*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d \varphi \supset^d \Box^d(\Diamond^d \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^d F10$

— nitpick[expect=none] — sledgehammer — none — no prf

**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf

**oops**

**lemma** *K4*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^d (\Box^d \varphi \supset^d \Box^d(\Box^d \varphi))) \longrightarrow \langle W,R,V \rangle, w \models^d F10$

— nitpick[expect=none] — sledgehammer — none — no prf

**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf

**oops**

**lemma** *K*:  $\forall w:W. \langle W,R,V \rangle, w \models^d F10$

— nitpick[expect=none] — sledgehammer — none — no prf

**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf

**oops**

**end**

— Summary of experiments: Nitpick: ctex=16 (with simp 10, without simp 6), none=74 (with simp 0, without simp 74), unknwn=70 (with simp 70, without simp 0) Sledgehammer: proof=33 (with simp 16, without simp 17), no prf=127 (with simp 64, without simp 63)

— No conflict

**end**

## 8.2 Tests with the deep embedding: semantic frame conditions

**theory** *PMLinHOL-deep-further-tests-2*

**imports** *PMLinHOL-deep-tests*

**begin**

— What is the weakest modal logic in which the following test formulas F1-F10 are provable?

— Test with semantic conditions

**abbreviation**(*input*) *refl* (*r*) **where**  $r \equiv \lambda R. \text{ reflexive } R$

**abbreviation**(*input*) *sym* (*s*) **where**  $s \equiv \lambda R. \text{ symmetric } R$

**abbreviation**(*input*) *tra* (*t*) **where**  $t \equiv \lambda R. \text{ transitive } R$

**consts**  $\varphi::PML \ \psi::PML$

**abbreviation**(*input*)  $F1 \equiv (\diamond^d(\diamond^d\varphi)) \supset^d \diamond^d\varphi$  — holds in K4

**abbreviation**(*input*)  $F2 \equiv (\diamond^d(\Box^d\varphi)) \supset^d \Box^d(\diamond^d\varphi)$  — holds in KB

**abbreviation**(*input*)  $F3 \equiv (\diamond^d(\Box^d\varphi)) \supset^d \Box^d\varphi$  — holds in KB4

**abbreviation**(*input*)  $F4 \equiv (\Box^d(\diamond^d(\Box^d(\diamond^d\varphi)))) \supset^d \Box^d(\diamond^d\varphi)$  — holds in KB/K4

**abbreviation**(*input*)  $F5 \equiv (\diamond^d(\varphi \wedge^d (\diamond^d\psi))) \supset^d ((\diamond^d\varphi) \wedge^d (\diamond^d\psi))$  — holds in K4

**abbreviation**(*input*)  $F6 \equiv ((\Box^d(\varphi \supset^d \psi)) \wedge^d (\diamond^d(\Box^d(\neg^d\psi)))) \supset^d \neg^d(\diamond^d\psi)$  — holds in KB4

**abbreviation**(*input*)  $F7 \equiv (\diamond^d\varphi) \supset^d (\Box^d(\varphi \vee^d \diamond^d\varphi))$  — holds in KB4

**abbreviation**(*input*)  $F8 \equiv (\diamond^d(\Box^d\varphi)) \supset^d (\varphi \vee^d \diamond^d\varphi)$  — holds in KT/KB

**abbreviation**(*input*)  $F9 \equiv ((\Box^d(\diamond^d\varphi)) \wedge^d (\Box^d(\diamond^d(\neg^d\varphi)))) \supset^d \diamond^d(\diamond^d\varphi)$  — holds in KT

**abbreviation**(*input*)  $F10 \equiv ((\Box^d(\varphi \supset^d \Box^d\psi)) \wedge^d (\Box^d(\diamond^d(\neg^d\psi)))) \supset^d \neg^d(\Box^d\psi)$  — holds in KT

**declare** *imp-cong*[*cong del*]

**experiment begin**

**lemma**  $S5: \forall w:W. r \ R \wedge s \ R \wedge t \ R \longrightarrow (\langle W,R,V \rangle, w \models^d F1)$

— nitpick[expect=none] — sledgehammer — none — proof

**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof

**by** *blast*

**lemma**  $S4: \forall w:W. r \ R \wedge t \ R \longrightarrow (\langle W,R,V \rangle, w \models^d F1)$

— nitpick[expect=none] — sledgehammer — none — proof

**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof

**by** *blast*

**lemma**  $KB4: \forall w:W. s \ R \wedge t \ R \longrightarrow (\langle W,R,V \rangle, w \models^d F1)$

— nitpick[expect=none] — sledgehammer — none — proof

**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof

**by** *blast*

**lemma**  $KTB: \forall w:W. r \ R \wedge s \ R \longrightarrow (\langle W,R,V \rangle, w \models^d F1)$

— nitpick[expect=genuine] — sledgehammer — ctex — no prf

**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf

**oops**

**lemma**  $KT: \forall w:W. r \ R \longrightarrow (\langle W,R,V \rangle, w \models^d F1)$

```

— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply simp — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma KB:  $\forall w:W. s R \longrightarrow (\langle W,R,V \rangle, w \models^d F1)$ 
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply simp — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma K4:  $\forall w:W. t R \longrightarrow (\langle W,R,V \rangle, w \models^d F1)$ 
— nitpick[expect=none] — sledgehammer — none — proof
apply simp — nitpick[expect=unknown] — sledgehammer — unkn — proof
by blast
lemma K:  $\forall w:W. (\langle W,R,V \rangle, w \models^d F1)$ 
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply simp — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
end

experiment begin
lemma S5:  $\forall w:W. r R \wedge s R \wedge t R \longrightarrow (\langle W,R,V \rangle, w \models^d F2)$ 
— nitpick[expect=none] — sledgehammer — none — proof
apply simp — nitpick[expect=unknown] — sledgehammer — unkn — proof
by blast
lemma S4:  $\forall w:W. r R \wedge t R \longrightarrow (\langle W,R,V \rangle, w \models^d F2)$ 
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply simp — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma KB4:  $\forall w:W. s R \wedge t R \longrightarrow (\langle W,R,V \rangle, w \models^d F2)$ 
— nitpick[expect=none] — sledgehammer — none — proof
apply simp — nitpick[expect=unknown] — sledgehammer — unkn — proof
by blast
lemma KTB:  $\forall w:W. r R \wedge s R \longrightarrow (\langle W,R,V \rangle, w \models^d F2)$ 
— nitpick[expect=none] — sledgehammer — none — proof
apply simp — nitpick[expect=unknown] — sledgehammer — unkn — proof
by meson
lemma KT:  $\forall w:W. r R \longrightarrow (\langle W,R,V \rangle, w \models^d F2)$ 
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply simp — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma KB:  $\forall w:W. s R \longrightarrow (\langle W,R,V \rangle, w \models^d F2)$ 
— nitpick[expect=none] — sledgehammer — none — proof
apply simp — nitpick[expect=unknown] — sledgehammer — unkn — proof
by blast
lemma K4:  $\forall w:W. t R \longrightarrow (\langle W,R,V \rangle, w \models^d F2)$ 
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply simp — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma K:  $\forall w:W. (\langle W,R,V \rangle, w \models^d F2)$ 
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply simp — nitpick[expect=unknown] — sledgehammer — unkn — no prf

```

**oops**  
**end**

**experiment begin**

**lemma**  $S5: \forall w:W. r R \wedge s R \wedge t R \longrightarrow (\langle W,R,V \rangle, w \models^d F3)$

— nitpick[expect=none] — sledgehammer — none — proof

**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof

**by** *blast*

**lemma**  $S4: \forall w:W. r R \wedge t R \longrightarrow (\langle W,R,V \rangle, w \models^d F3)$

— nitpick[expect=genuine] — sledgehammer — ctex — no prf

**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf

**oops**

**lemma**  $KB4: \forall w:W. s R \wedge t R \longrightarrow (\langle W,R,V \rangle, w \models^d F3)$

— nitpick[expect=none] — sledgehammer — none — proof

**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof

**by** *blast*

**lemma**  $KTB: \forall w:W. r R \wedge s R \longrightarrow (\langle W,R,V \rangle, w \models^d F3)$

— nitpick[expect=genuine] — sledgehammer — ctex — no prf

**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf

**oops**

**lemma**  $KT: \forall w:W. r R \longrightarrow (\langle W,R,V \rangle, w \models^d F3)$

— nitpick[expect=genuine] — sledgehammer — ctex — no prf

**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf

**oops**

**lemma**  $KB: \forall w:W. s R \longrightarrow (\langle W,R,V \rangle, w \models^d F3)$

— nitpick[expect=genuine] — sledgehammer — ctex — no prf

**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf

**oops**

**lemma**  $K4: \forall w:W. t R \longrightarrow (\langle W,R,V \rangle, w \models^d F3)$

— nitpick[expect=genuine] — sledgehammer — ctex — no prf

**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf

**oops**

**lemma**  $K: \forall w:W. (\langle W,R,V \rangle, w \models^d F3)$

— nitpick[expect=genuine] — sledgehammer — ctex — no prf

**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf

**oops**

**end**

**experiment begin**

**lemma**  $S5: \forall w:W. r R \wedge s R \wedge t R \longrightarrow (\langle W,R,V \rangle, w \models^d F4)$

— nitpick[expect=none] — sledgehammer — none — proof

**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof

**by** *meson*

**lemma**  $S4: \forall w:W. r R \wedge t R \longrightarrow (\langle W,R,V \rangle, w \models^d F4)$

— nitpick[expect=none] — sledgehammer — none — no prf

**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof

**by** *meson*

**lemma**  $KB4: \forall w:W. s R \wedge t R \longrightarrow (\langle W,R,V \rangle, w \models^d F4)$

— nitpick[expect=none] — sledgehammer — none — proof

**apply simp** — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**by meson**  
**lemma KTB**:  $\forall w:W. r R \wedge s R \longrightarrow (\langle W,R,V \rangle, w \models^d F4)$   
— nitpick[expect=none] — sledgehammer — none — proof  
**apply simp** — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**by meson**  
**lemma KT**:  $\forall w:W. r R \longrightarrow (\langle W,R,V \rangle, w \models^d F4)$   
— nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply simp** — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**  
**lemma KB**:  $\forall w:W. s R \longrightarrow (\langle W,R,V \rangle, w \models^d F4)$   
— nitpick[expect=none] — sledgehammer — none — proof  
**apply simp** — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**by blast**  
**lemma K4**:  $\forall w:W. t R \longrightarrow (\langle W,R,V \rangle, w \models^d F4)$   
— nitpick[expect=none] — sledgehammer — none — no prf  
**apply simp** — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**by meson**  
**lemma K**:  $\forall w:W. (\langle W,R,V \rangle, w \models^d F4)$   
— nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply simp** — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**  
**end**

#### experiment begin

**lemma S5**:  $\forall w:W. r R \wedge s R \wedge t R \longrightarrow (\langle W,R,V \rangle, w \models^d F5)$   
— nitpick[expect=none] — sledgehammer — none — no prf  
**apply simp** — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**by blast**  
**lemma S4**:  $\forall w:W. r R \wedge t R \longrightarrow (\langle W,R,V \rangle, w \models^d F5)$   
— nitpick[expect=none] — sledgehammer — none — no prf  
**apply simp** — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**by blast**  
**lemma KB4**:  $\forall w:W. s R \wedge t R \longrightarrow (\langle W,R,V \rangle, w \models^d F5)$   
— nitpick[expect=none] — sledgehammer — none — proof  
**apply simp** — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**by blast**  
**lemma KTB**:  $\forall w:W. r R \wedge s R \longrightarrow (\langle W,R,V \rangle, w \models^d F5)$   
— nitpick[expect=none] — sledgehammer — none — no prf  
**apply simp** — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**  
**lemma KT**:  $\forall w:W. r R \longrightarrow (\langle W,R,V \rangle, w \models^d F5)$   
— nitpick[expect=none] — sledgehammer — none — no prf  
**apply simp** — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**  
**lemma KB**:  $\forall w:W. s R \longrightarrow (\langle W,R,V \rangle, w \models^d F5)$   
— nitpick[expect=none] — sledgehammer — none — no prf  
**apply simp** — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**

**lemma**  $K4$ :  $\forall w: W. \text{t } R \longrightarrow (\langle W, R, V \rangle, w \models^d F5)$   
 — nitpick[expect=none] — sledgehammer — none — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**by** *blast*

**lemma**  $K$ :  $\forall w: W. (\langle W, R, V \rangle, w \models^d F5)$   
 — nitpick[expect=none] — sledgehammer — none — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**  
**end**

**experiment begin**

**lemma**  $S5$ :  $\forall w: W. \text{r } R \wedge \text{s } R \wedge \text{t } R \longrightarrow (\langle W, R, V \rangle, w \models^d F6)$   
 — nitpick[expect=none] — sledgehammer — none — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**by** *blast*

**lemma**  $S4$ :  $\forall w: W. \text{r } R \wedge \text{t } R \longrightarrow (\langle W, R, V \rangle, w \models^d F6)$   
 — nitpick[expect=none] — sledgehammer — none — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**

**lemma**  $KB4$ :  $\forall w: W. \text{s } R \wedge \text{t } R \longrightarrow (\langle W, R, V \rangle, w \models^d F6)$   
 — nitpick[expect=none] — sledgehammer — none — proof  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**by** *blast*

**lemma**  $KTB$ :  $\forall w: W. \text{r } R \wedge \text{s } R \longrightarrow (\langle W, R, V \rangle, w \models^d F6)$   
 — nitpick[expect=none] — sledgehammer — none — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**

**lemma**  $KT$ :  $\forall w: W. \text{r } R \longrightarrow (\langle W, R, V \rangle, w \models^d F6)$   
 — nitpick[expect=none] — sledgehammer — none — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**

**lemma**  $KB$ :  $\forall w: W. \text{s } R \longrightarrow (\langle W, R, V \rangle, w \models^d F6)$   
 — nitpick[expect=none] — sledgehammer — none — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**

**lemma**  $K4$ :  $\forall w: W. \text{t } R \longrightarrow (\langle W, R, V \rangle, w \models^d F6)$   
 — nitpick[expect=none] — sledgehammer — none — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**

**lemma**  $K$ :  $\forall w: W. (\langle W, R, V \rangle, w \models^d F6)$   
 — nitpick[expect=none] — sledgehammer — none — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**  
**end**

**experiment begin**

**lemma**  $S5$ :  $\forall w: W. \text{r } R \wedge \text{s } R \wedge \text{t } R \longrightarrow (\langle W, R, V \rangle, w \models^d F7)$   
 — nitpick[expect=none] — sledgehammer — none — proof  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof

**by** *blast*  
**lemma**  $S_4$ :  $\forall w:W. \text{r } R \wedge \text{t } R \longrightarrow (\langle W,R,V \rangle, w \models^d F\gamma)$   
— nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**  
**lemma**  $KB_4$ :  $\forall w:W. \text{s } R \wedge \text{t } R \longrightarrow (\langle W,R,V \rangle, w \models^d F\gamma)$   
— nitpick[expect=none] — sledgehammer — none — proof  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**by** *blast*  
**lemma**  $KTB$ :  $\forall w:W. \text{r } R \wedge \text{s } R \longrightarrow (\langle W,R,V \rangle, w \models^d F\gamma)$   
— nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**  
**lemma**  $KT$ :  $\forall w:W. \text{r } R \longrightarrow (\langle W,R,V \rangle, w \models^d F\gamma)$   
— nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**  
**lemma**  $KB$ :  $\forall w:W. \text{s } R \longrightarrow (\langle W,R,V \rangle, w \models^d F\gamma)$   
— nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**  
**lemma**  $K_4$ :  $\forall w:W. \text{t } R \longrightarrow (\langle W,R,V \rangle, w \models^d F\gamma)$   
— nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**  
**lemma**  $K$ :  $\forall w:W. (\langle W,R,V \rangle, w \models^d F\gamma)$   
— nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**  
**end**

### experiment begin

**lemma**  $S5$ :  $\forall w:W. \text{r } R \wedge \text{s } R \wedge \text{t } R \longrightarrow (\langle W,R,V \rangle, w \models^d F8)$   
— nitpick[expect=none] — sledgehammer — none — proof  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**by** *blast*  
**lemma**  $S_4$ :  $\forall w:W. \text{r } R \wedge \text{t } R \longrightarrow (\langle W,R,V \rangle, w \models^d F8)$   
— nitpick[expect=none] — sledgehammer — none — proof  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**by** *blast*  
**lemma**  $KB_4$ :  $\forall w:W. \text{s } R \wedge \text{t } R \longrightarrow (\langle W,R,V \rangle, w \models^d F8)$   
— nitpick[expect=none] — sledgehammer — none — proof  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**by** *blast*  
**lemma**  $KTB$ :  $\forall w:W. \text{r } R \wedge \text{s } R \longrightarrow (\langle W,R,V \rangle, w \models^d F8)$   
— nitpick[expect=none] — sledgehammer — none — proof  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**by** *blast*  
**lemma**  $KT$ :  $\forall w:W. \text{r } R \longrightarrow (\langle W,R,V \rangle, w \models^d F8)$

```

— nitpick[expect=none] — sledgehammer — none — proof
apply simp — nitpick[expect=unknown] — sledgehammer — unkn — proof
by blast
lemma KB:  $\forall w:W. s R \longrightarrow (\langle W,R,V \rangle, w \models^d F8)$ 
— nitpick[expect=none] — sledgehammer — none — proof
apply simp — nitpick[expect=unknown] — sledgehammer — unkn — proof
by blast
lemma K4:  $\forall w:W. t R \longrightarrow (\langle W,R,V \rangle, w \models^d F8)$ 
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply simp — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma K:  $\forall w:W. (\langle W,R,V \rangle, w \models^d F8)$ 
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply simp — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
end

```

#### experiment begin

```

lemma S5:  $\forall w:W. r R \wedge s R \wedge t R \longrightarrow (\langle W,R,V \rangle, w \models^d F9)$ 
— nitpick[expect=none] — sledgehammer — none — proof
apply simp — nitpick[expect=unknown] — sledgehammer — unkn — proof
by blast
lemma S4:  $\forall w:W. r R \wedge t R \longrightarrow (\langle W,R,V \rangle, w \models^d F9)$ 
— nitpick[expect=none] — sledgehammer — none — proof
apply simp — nitpick[expect=unknown] — sledgehammer — unkn — proof
by blast
lemma KB4:  $\forall w:W. s R \wedge t R \longrightarrow (\langle W,R,V \rangle, w \models^d F9)$ 
— nitpick[expect=none] — sledgehammer — none — no prf
apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma KTB:  $\forall w:W. r R \wedge s R \longrightarrow (\langle W,R,V \rangle, w \models^d F9)$ 
— nitpick[expect=none] — sledgehammer — none — proof
apply simp — nitpick[expect=unknown] — sledgehammer — unkn — proof
by blast
lemma KT:  $\forall w:W. r R \longrightarrow (\langle W,R,V \rangle, w \models^d F9)$ 
— nitpick[expect=none] — sledgehammer — none — proof
apply simp — nitpick[expect=unknown] — sledgehammer — unkn — proof
by blast
lemma KB:  $\forall w:W. s R \longrightarrow (\langle W,R,V \rangle, w \models^d F9)$ 
— nitpick[expect=none] — sledgehammer — none — no prf
apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma K4:  $\forall w:W. t R \longrightarrow (\langle W,R,V \rangle, w \models^d F9)$ 
— nitpick[expect=none] — sledgehammer — none — no prf
apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma K:  $\forall w:W. (\langle W,R,V \rangle, w \models^d F9)$ 
— nitpick[expect=none] — sledgehammer — none — no prf
apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf

```

**oops**  
**end**

**experiment begin**

**lemma**  $S5: \forall w:W. r R \wedge s R \wedge t R \longrightarrow (\langle W,R,V \rangle, w \models^d F10)$

— nitpick[expect=none] — sledgehammer — none — proof

**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof

**by** *blast*

**lemma**  $S4: \forall w:W. r R \wedge t R \longrightarrow (\langle W,R,V \rangle, w \models^d F10)$

— nitpick[expect=none] — sledgehammer — none — proof

**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof

**by** *blast*

**lemma**  $KB4: \forall w:W. s R \wedge t R \longrightarrow (\langle W,R,V \rangle, w \models^d F10)$

— nitpick[expect=none] — sledgehammer — none — no prf

**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf

**oops**

**lemma**  $KTB: \forall w:W. r R \wedge s R \longrightarrow (\langle W,R,V \rangle, w \models^d F10)$

— nitpick[expect=none] — sledgehammer — none — proof

**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof

**by** *blast*

**lemma**  $KT: \forall w:W. r R \longrightarrow (\langle W,R,V \rangle, w \models^d F10)$

— nitpick[expect=none] — sledgehammer — none — proof

**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof

**by** *blast*

**lemma**  $KB: \forall w:W. s R \longrightarrow (\langle W,R,V \rangle, w \models^d F10)$

— nitpick[expect=none] — sledgehammer — none — no prf

**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf

**oops**

**lemma**  $K4: \forall w:W. t R \longrightarrow (\langle W,R,V \rangle, w \models^d F10)$

— nitpick[expect=none] — sledgehammer — none — no prf

**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf

**oops**

**lemma**  $K: \forall w:W. (\langle W,R,V \rangle, w \models^d F10)$

— nitpick[expect=none] — sledgehammer — none — no prf

**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf

**oops**

**end**

— Summary of experiments: Nitpick: ctex=32 (with simp 8, without simp 24), none=56 (with simp 0, without simp 56), unknwn=72 (with simp 72, without simp 0) Sledgehammer: proof=70 (with simp 38, without simp 32), no prf=90 (with simp 42, without simp 48)

— No conflicts

**end**

### 8.3 Tests with the (maximal) shallow embedding: axiom schemata

**theory** *PMLinHOL-shallow-further-tests-1* **imports** *PMLinHOL-shallow-tests*

**begin**

- What is the weakest modal logic in which the following test formulas F1-F10 are provable?
- Test with schematic axioms

**consts**  $\varphi::\sigma \ \psi::\sigma$

**abbreviation**(*input*)  $F1 \equiv (\diamond^s(\diamond^s\varphi)) \supset^s \diamond^s\varphi$  — holds in K4

**abbreviation**(*input*)  $F2 \equiv (\diamond^s(\Box^s\varphi)) \supset^s \Box^s(\diamond^s\varphi)$  — holds in KB

**abbreviation**(*input*)  $F3 \equiv (\diamond^s(\Box^s\varphi)) \supset^s \Box^s\varphi$  — holds in KB4

**abbreviation**(*input*)  $F4 \equiv (\Box^s(\diamond^s(\Box^s(\diamond^s\varphi)))) \supset^s \Box^s(\diamond^s\varphi)$  — holds in KB/K4

**abbreviation**(*input*)  $F5 \equiv (\diamond^s(\varphi \wedge^s (\diamond^s\psi))) \supset^s ((\diamond^s\varphi) \wedge^s (\diamond^s\psi))$  — holds in K4

**abbreviation**(*input*)  $F6 \equiv ((\Box^s(\varphi \supset^s \psi)) \wedge^s (\Box^s(\Box^s(\neg^s\psi)))) \supset^s \neg^s(\diamond^s\psi)$  — holds in KB4

**abbreviation**(*input*)  $F7 \equiv (\diamond^s\varphi) \supset^s (\Box^s(\varphi \vee^s \diamond^s\varphi))$  — holds in KB4

**abbreviation**(*input*)  $F8 \equiv (\diamond^s(\Box^s\varphi)) \supset^s (\varphi \vee^s \diamond^s\varphi)$  — holds in KT/KB

**abbreviation**(*input*)  $F9 \equiv ((\Box^s(\diamond^s\varphi)) \wedge^s (\Box^s(\diamond^s(\neg^s\varphi)))) \supset^s \diamond^s(\diamond^s\varphi)$  — holds in KT

**abbreviation**(*input*)  $F10 \equiv ((\Box^s(\varphi \supset^s \Box^s\psi)) \wedge^s (\Box^s(\diamond^s(\neg^s\psi)))) \supset^s \neg^s(\Box^s\psi)$  — holds in KT

**declare** *imp-cong*[*cong del*]

**experiment begin**

**lemma**  $S5$ :  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^s \varphi \supset^s \Box^s(\diamond^s\varphi)) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V \rangle, w \models^s F1$

— nitpick[expect=unknown] — sledgehammer — unkn — no prf

**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof

**by** (*smt* (*verit*) *NegS-def*)

**lemma**  $S4$ :  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V \rangle, w \models^s F1$

— nitpick[expect=unknown] — sledgehammer — unkn — no prf

**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof

**by** (*metis* *NegS-def*)

**lemma**  $KB4$ :  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^s \varphi \supset^s \Box^s(\diamond^s\varphi)) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V \rangle, w \models^s F1$

— nitpick[expect=unknown] — sledgehammer — unkn — no prf

**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof

**by** (*smt* (*verit*) *NegS-def*)

**lemma**  $KTB$ :  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^s \varphi \supset^s \Box^s(\diamond^s\varphi)) \longrightarrow \langle W,R,V \rangle, w \models^s F1$

— nitpick[expect=unknown] — sledgehammer — unkn — no prf

**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf

**oops**

**lemma**  $KT$ :  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s\varphi) \supset^s \varphi) \longrightarrow \langle W,R,V \rangle, w \models^s F1$

— nitpick[expect=unknown] — sledgehammer — unkn — no prf

**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf

**oops**

**lemma**  $KB$ :  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^s \varphi \supset^s \Box^s(\diamond^s\varphi)) \longrightarrow \langle W,R,V \rangle, w \models^s F1$

— nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**apply simp** — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**  
**lemma K4**:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s \varphi) \supset^s \Box^s(\Box^s \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^s F1$   
 — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**apply simp** — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**by** (*metis NegS-def*)  
**lemma K**:  $\forall w:W. \langle W,R,V \rangle, w \models^s F1$   
 — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply simp** — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**oops**  
**end**

**experiment begin**  
**lemma S5**:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s \varphi) \supset^s \varphi) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^s \varphi \supset^s \Box^s(\Diamond^s \varphi)) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s \varphi) \supset^s \Box^s(\Box^s \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^s F2$   
 — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**apply simp** — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**by** (*smt (verit) NegS-def*)  
**lemma S4**:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s \varphi) \supset^s \varphi) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s \varphi) \supset^s \Box^s(\Box^s \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^s F2$   
 — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**apply simp** — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**  
**lemma KB4**:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^s \varphi \supset^s \Box^s(\Diamond^s \varphi)) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s \varphi) \supset^s \Box^s(\Box^s \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^s F2$   
 — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**apply simp** — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**by** (*smt (verit) NegS-def*)  
**lemma KTB**:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s \varphi) \supset^s \varphi) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^s \varphi \supset^s \Box^s(\Diamond^s \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^s F2$   
 — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**apply simp** — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**by** (*smt (verit) NegS-def*)  
**lemma KT**:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s \varphi) \supset^s \varphi) \longrightarrow \langle W,R,V \rangle, w \models^s F2$   
 — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**apply simp** — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**  
**lemma KB**:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^s \varphi \supset^s \Box^s(\Diamond^s \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^s F2$   
 — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**apply simp** — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**by** (*smt (verit) NegS-def*)  
**lemma K4**:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s \varphi) \supset^s \Box^s(\Box^s \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^s F2$   
 — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**apply simp** — nitpick[expect=unknown] — sledgehammer — unkn — no prf

**oops**  
**lemma**  $K: \forall w: W. \langle W, R, V \rangle, w \models^s F2$   
— nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**oops**  
**end**

**experiment begin**  
**lemma**  $S5: \forall w: W. (\forall \varphi. \langle W, R, V \rangle, w \models^s (\Box^s \varphi) \supset^s \varphi) \wedge (\forall \varphi. \langle W, R, V \rangle, w \models^s \varphi \supset^s \Box^s(\Diamond^s \varphi)) \wedge (\forall \varphi. \langle W, R, V \rangle, w \models^s (\Box^s \varphi) \supset^s \Box^s(\Box^s \varphi)) \longrightarrow \langle W, R, V \rangle, w \models^s F3$   
— nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**  
**lemma**  $S4: \forall w: W. (\forall \varphi. \langle W, R, V \rangle, w \models^s (\Box^s \varphi) \supset^s \varphi) \wedge (\forall \varphi. \langle W, R, V \rangle, w \models^s (\Box^s \varphi) \supset^s \Box^s(\Box^s \varphi)) \longrightarrow \langle W, R, V \rangle, w \models^s F3$   
— nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**  
**lemma**  $KB4: \forall w: W. (\forall \varphi. \langle W, R, V \rangle, w \models^s \varphi \supset^s \Box^s(\Diamond^s \varphi)) \wedge (\forall \varphi. \langle W, R, V \rangle, w \models^s (\Box^s \varphi) \supset^s \Box^s(\Box^s \varphi)) \longrightarrow \langle W, R, V \rangle, w \models^s F3$   
— nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**  
**lemma**  $KTB: \forall w: W. (\forall \varphi. \langle W, R, V \rangle, w \models^s (\Box^s \varphi) \supset^s \varphi) \wedge (\forall \varphi. \langle W, R, V \rangle, w \models^s \varphi \supset^s \Box^s(\Diamond^s \varphi)) \longrightarrow \langle W, R, V \rangle, w \models^s F3$   
— nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**  
**lemma**  $KT: \forall w: W. (\forall \varphi. \langle W, R, V \rangle, w \models^s (\Box^s \varphi) \supset^s \varphi) \longrightarrow \langle W, R, V \rangle, w \models^s F3$   
— nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**  
**lemma**  $KB: \forall w: W. (\forall \varphi. \langle W, R, V \rangle, w \models^s \varphi \supset^s \Box^s(\Diamond^s \varphi)) \longrightarrow \langle W, R, V \rangle, w \models^s F3$   
— nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**  
**lemma**  $K4: \forall w: W. (\forall \varphi. \langle W, R, V \rangle, w \models^s (\Box^s \varphi) \supset^s \Box^s(\Box^s \varphi)) \longrightarrow \langle W, R, V \rangle, w \models^s F3$   
— nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**  
**lemma**  $K: \forall w: W. \langle W, R, V \rangle, w \models^s F3$   
— nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**oops**  
**end**

**experiment begin**

**lemma S5:**  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s \varphi) \supset^s \varphi) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^s \varphi \supset^s \Box^s(\Diamond^s \varphi)) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s \varphi) \supset^s \Box^s(\Box^s \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^s F4$   
— nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**apply simp** — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**

**lemma S4:**  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s \varphi) \supset^s \varphi) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s \varphi) \supset^s \Box^s(\Box^s \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^s F4$   
— nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**apply simp** — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**

**lemma KB4:**  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^s \varphi \supset^s \Box^s(\Diamond^s \varphi)) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s \varphi) \supset^s \Box^s(\Box^s \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^s F4$   
— nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**apply simp** — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**

**lemma KTB:**  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s \varphi) \supset^s \varphi) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^s \varphi \supset^s \Box^s(\Diamond^s \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^s F4$   
— nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**apply simp** — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**

**lemma KT:**  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s \varphi) \supset^s \varphi) \longrightarrow \langle W,R,V \rangle, w \models^s F4$   
— nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**apply simp** — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**

**lemma KB:**  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^s \varphi \supset^s \Box^s(\Diamond^s \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^s F4$   
— nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**apply simp** — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**

**lemma K4:**  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s \varphi) \supset^s \Box^s(\Box^s \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^s F4$   
— nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**apply simp** — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**

**lemma K:**  $\forall w:W. \langle W,R,V \rangle, w \models^s F4$   
— nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply simp** — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**oops**

**end**

### experiment begin

**lemma S5:**  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s \varphi) \supset^s \varphi) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^s \varphi \supset^s \Box^s(\Diamond^s \varphi)) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s \varphi) \supset^s \Box^s(\Box^s \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^s F5$   
— nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**apply simp** — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**by (smt (verit) NegS-def)**

**lemma S4:**  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s \varphi) \supset^s \varphi) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s \varphi) \supset^s \Box^s(\Box^s \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^s F5$   
— nitpick[expect=unknown] — sledgehammer — unkn — no prf

**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**by** (*smt (verit) NegS-def*)  
**lemma** *KB4*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^s \varphi \supset^s \Box^s(\Diamond^s \varphi)) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s \varphi) \supset^s \Box^s(\Box^s \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^s F5$   
— nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**by** (*smt (verit) NegS-def*)  
**lemma** *KTB*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s \varphi) \supset^s \varphi) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^s \varphi \supset^s \Box^s(\Diamond^s \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^s F5$   
— nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**  
**lemma** *KT*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s \varphi) \supset^s \varphi) \longrightarrow \langle W,R,V \rangle, w \models^s F5$   
— nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**  
**lemma** *KB*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^s \varphi \supset^s \Box^s(\Diamond^s \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^s F5$   
— nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**  
**lemma** *K4*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s \varphi) \supset^s \Box^s(\Box^s \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^s F5$   
— nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**by** (*smt (verit, del-insts) NegS-def*)  
**lemma** *K*:  $\forall w:W. \langle W,R,V \rangle, w \models^s F5$   
— nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**oops**  
**end**

### experiment begin

**lemma** *S5*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s \varphi) \supset^s \varphi) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^s \varphi \supset^s \Box^s(\Diamond^s \varphi)) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s \varphi) \supset^s \Box^s(\Box^s \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^s F6$   
— nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**  
**lemma** *S4*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s \varphi) \supset^s \varphi) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s \varphi) \supset^s \Box^s(\Box^s \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^s F6$   
— nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**  
**lemma** *KB4*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^s \varphi \supset^s \Box^s(\Diamond^s \varphi)) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s \varphi) \supset^s \Box^s(\Box^s \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^s F6$   
— nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**  
**lemma** *KTB*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s \varphi) \supset^s \varphi) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^s$

$\varphi \supset^s \Box^s(\Diamond^s \varphi) \longrightarrow \langle W, R, V \rangle, w \models^s F6$   
 — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**apply simp** — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**

**lemma KT**:  $\forall w: W. (\forall \varphi. \langle W, R, V \rangle, w \models^s (\Box^s \varphi) \supset^s \varphi) \longrightarrow \langle W, R, V \rangle, w \models^s F6$   
 — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**apply simp** — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**

**lemma KB**:  $\forall w: W. (\forall \varphi. \langle W, R, V \rangle, w \models^s \varphi \supset^s \Box^s(\Diamond^s \varphi)) \longrightarrow \langle W, R, V \rangle, w \models^s F6$   
 — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**apply simp** — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**

**lemma K4**:  $\forall w: W. (\forall \varphi. \langle W, R, V \rangle, w \models^s (\Box^s \varphi) \supset^s \Box^s(\Box^s \varphi)) \longrightarrow \langle W, R, V \rangle, w \models^s F6$   
 — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**apply simp** — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**

**lemma K**:  $\forall w: W. \langle W, R, V \rangle, w \models^s F6$   
 — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply simp** — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**oops**

**end**

**experiment begin**

**lemma S5**:  $\forall w: W. (\forall \varphi. \langle W, R, V \rangle, w \models^s (\Box^s \varphi) \supset^s \varphi) \wedge (\forall \varphi. \langle W, R, V \rangle, w \models^s \varphi \supset^s \Box^s(\Diamond^s \varphi)) \wedge (\forall \varphi. \langle W, R, V \rangle, w \models^s (\Box^s \varphi) \supset^s \Box^s(\Box^s \varphi)) \longrightarrow \langle W, R, V \rangle, w \models^s F7$   
 — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**apply simp** — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**

**lemma S4**:  $\forall w: W. (\forall \varphi. \langle W, R, V \rangle, w \models^s (\Box^s \varphi) \supset^s \varphi) \wedge (\forall \varphi. \langle W, R, V \rangle, w \models^s (\Box^s \varphi) \supset^s \Box^s(\Box^s \varphi)) \longrightarrow \langle W, R, V \rangle, w \models^s F7$   
 — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**apply simp** — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**

**lemma KB4**:  $\forall w: W. (\forall \varphi. \langle W, R, V \rangle, w \models^s \varphi \supset^s \Box^s(\Diamond^s \varphi)) \wedge (\forall \varphi. \langle W, R, V \rangle, w \models^s (\Box^s \varphi) \supset^s \Box^s(\Box^s \varphi)) \longrightarrow \langle W, R, V \rangle, w \models^s F7$   
 — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**apply simp** — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**

**lemma KTB**:  $\forall w: W. (\forall \varphi. \langle W, R, V \rangle, w \models^s (\Box^s \varphi) \supset^s \varphi) \wedge (\forall \varphi. \langle W, R, V \rangle, w \models^s \varphi \supset^s \Box^s(\Diamond^s \varphi)) \longrightarrow \langle W, R, V \rangle, w \models^s F7$   
 — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**apply simp** — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**

**lemma KT**:  $\forall w: W. (\forall \varphi. \langle W, R, V \rangle, w \models^s (\Box^s \varphi) \supset^s \varphi) \longrightarrow \langle W, R, V \rangle, w \models^s F7$   
 — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**apply simp** — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**

**lemma** *KB*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^s \varphi \supset^s \Box^s(\Diamond^s \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^s F7$

— nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**

**lemma** *K4*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s \varphi) \supset^s \Box^s(\Box^s \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^s F7$

— nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**

**lemma** *K*:  $\forall w:W. \langle W,R,V \rangle, w \models^s F7$

— nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**oops**

**end**

**experiment begin**

**lemma** *S5*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s \varphi) \supset^s \varphi) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^s \varphi \supset^s \Box^s(\Diamond^s \varphi)) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s \varphi) \supset^s \Box^s(\Box^s \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^s F8$

— nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**by** (*metis NegS-def*)

**lemma** *S4*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s \varphi) \supset^s \varphi) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s \varphi) \supset^s \Box^s(\Box^s \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^s F8$

— nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**

**lemma** *KB4*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^s \varphi \supset^s \Box^s(\Diamond^s \varphi)) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s \varphi) \supset^s \Box^s(\Box^s \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^s F8$

— nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**by** (*metis NegS-def*)

**lemma** *KTB*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s \varphi) \supset^s \varphi) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^s \varphi \supset^s \Box^s(\Diamond^s \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^s F8$

— nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**by** (*metis NegS-def*)

**lemma** *KT*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s \varphi) \supset^s \varphi) \longrightarrow \langle W,R,V \rangle, w \models^s F8$

— nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**

**lemma** *KB*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^s \varphi \supset^s \Box^s(\Diamond^s \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^s F8$

— nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**by** (*metis NegS-def*)

**lemma** *K4*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s \varphi) \supset^s \Box^s(\Box^s \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^s F8$

— nitpick[expect=unknown] — sledgehammer — unkn — no prf

**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**oops**  
**lemma** *K*:  $\forall w:W. \langle W,R,V \rangle, w \models^s F8$   
— nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**oops**  
**end**

**experiment begin**  
**lemma** *S5*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s \varphi) \supset^s \varphi) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^s \varphi \supset^s \Box^s(\Diamond^s \varphi)) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s \varphi) \supset^s \Box^s(\Box^s \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^s F9$   
— nitpick[expect=unknown] — sledgehammer — unkn — proof  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**by** (*metis NegS-def*)  
**lemma** *S4*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s \varphi) \supset^s \varphi) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s \varphi) \supset^s \Box^s(\Box^s \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^s F9$   
— nitpick[expect=unknown] — sledgehammer — unkn — proof  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**by** (*metis NegS-def*)  
**lemma** *KB4*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^s \varphi \supset^s \Box^s(\Diamond^s \varphi)) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s \varphi) \supset^s \Box^s(\Box^s \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^s F9$   
— nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**oops**  
**lemma** *KTB*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s \varphi) \supset^s \varphi) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^s \varphi \supset^s \Box^s(\Diamond^s \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^s F9$   
— nitpick[expect=unknown] — sledgehammer — unkn — proof  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**by** (*metis NegS-def*)  
**lemma** *KT*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s \varphi) \supset^s \varphi) \longrightarrow \langle W,R,V \rangle, w \models^s F9$   
— nitpick[expect=unknown] — sledgehammer — unkn — proof  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**by** (*metis NegS-def*)  
**lemma** *KB*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^s \varphi \supset^s \Box^s(\Diamond^s \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^s F9$   
— nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**oops**  
**lemma** *K4*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s \varphi) \supset^s \Box^s(\Box^s \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^s F9$   
— nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**oops**  
**lemma** *K*:  $\forall w:W. \langle W,R,V \rangle, w \models^s F9$   
— nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**oops**  
**end**

**experiment begin**

**lemma** *S5*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s \varphi) \supset^s \varphi) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^s \varphi \supset^s \Box^s(\Diamond^s \varphi)) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s \varphi) \supset^s \Box^s(\Box^s \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^s F10$

— nitpick[expect=unknown] — sledgehammer — unkn — proof

**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof

**by** (*metis NegS-def*)

**lemma** *S4*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s \varphi) \supset^s \varphi) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s \varphi) \supset^s \Box^s(\Box^s \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^s F10$

— nitpick[expect=unknown] — sledgehammer — unkn — proof

**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof

**by** (*metis NegS-def*)

**lemma** *KB4*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^s \varphi \supset^s \Box^s(\Diamond^s \varphi)) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s \varphi) \supset^s \Box^s(\Box^s \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^s F10$

— nitpick[expect=genuine] — sledgehammer — ctex — no prf

**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf

**oops**

**lemma** *KTB*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s \varphi) \supset^s \varphi) \wedge (\forall \varphi. \langle W,R,V \rangle, w \models^s \varphi \supset^s \Box^s(\Diamond^s \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^s F10$

— nitpick[expect=unknown] — sledgehammer — unkn — proof

**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof

**by** (*metis (lifting) BoxS-def NegS-def*)

**lemma** *KT*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s \varphi) \supset^s \varphi) \longrightarrow \langle W,R,V \rangle, w \models^s F10$

— nitpick[expect=unknown] — sledgehammer — unkn — proof

**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof

**by** (*metis (lifting) BoxS-def NegS-def*)

**lemma** *KB*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^s \varphi \supset^s \Box^s(\Diamond^s \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^s F10$

— nitpick[expect=genuine] — sledgehammer — ctex — no prf

**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf

**oops**

**lemma** *K4*:  $\forall w:W. (\forall \varphi. \langle W,R,V \rangle, w \models^s (\Box^s \varphi) \supset^s \Box^s(\Box^s \varphi)) \longrightarrow \langle W,R,V \rangle, w \models^s F10$

— nitpick[expect=genuine] — sledgehammer — ctex — no prf

**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf

**oops**

**lemma** *K*:  $\forall w:W. \langle W,R,V \rangle, w \models^s F10$

— nitpick[expect=genuine] — sledgehammer — ctex — no prf

**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf

**oops**

**end**

— Summary of experiments: Nitpick: ctex=32 (with simp 16, without simp 16), none=0, unknwn=128 (with simp 64, without simp 64) Sledgehammer: proof=32 (with simp 24, without simp 8), no prf=128 (with simp 56, without simp 72)

— No conflict

**end**

## 8.4 Tests with the (maximal) shallow embedding: semantic frame conditions

**theory** *PMLinHOL-shallow-further-tests-2*

**imports** *PMLinHOL-shallow-tests*

**begin**

— What is the weakest modal logic in which the following test formulas F1-F10 are provable?

— Test with semantic conditions

**abbreviation**(*input*) *refl* (*r*) **where**  $r \equiv \lambda R. \text{ reflexive } R$

**abbreviation**(*input*) *sym* (*s*) **where**  $s \equiv \lambda R. \text{ symmetric } R$

**abbreviation**(*input*) *tra* (*t*) **where**  $t \equiv \lambda R. \text{ transitive } R$

**consts**  $\varphi::\sigma$   $\psi::\sigma$

**abbreviation**(*input*)  $F1 \equiv (\Diamond^s(\Diamond^s\varphi)) \supset^s \Diamond^s\varphi$  — holds in K4

**abbreviation**(*input*)  $F2 \equiv (\Diamond^s(\Box^s\varphi)) \supset^s \Box^s(\Diamond^s\varphi)$  — holds in KB

**abbreviation**(*input*)  $F3 \equiv (\Diamond^s(\Box^s\varphi)) \supset^s \Box^s\varphi$  — holds in KB4

**abbreviation**(*input*)  $F4 \equiv (\Box^s(\Diamond^s(\Box^s(\Diamond^s\varphi)))) \supset^s \Box^s(\Diamond^s\varphi)$  — holds in KB

**abbreviation**(*input*)  $F5 \equiv (\Diamond^s(\varphi \wedge^s (\Diamond^s\psi))) \supset^s ((\Diamond^s\varphi) \wedge^s (\Diamond^s\psi))$  — holds in K4

**abbreviation**(*input*)  $F6 \equiv ((\Box^s(\varphi \supset^s \psi)) \wedge^s (\Diamond^s(\Box^s(\neg^s\psi)))) \supset^s \neg^s(\Diamond^s\psi)$  — holds in KB4

**abbreviation**(*input*)  $F7 \equiv (\Diamond^s\varphi) \supset^s (\Box^s(\varphi \vee^s \Diamond^s\varphi))$  — holds in KB4

**abbreviation**(*input*)  $F8 \equiv (\Diamond^s(\Box^s\varphi)) \supset^s (\varphi \vee^s \Diamond^s\varphi)$  — holds in KT and in KB

**abbreviation**(*input*)  $F9 \equiv ((\Box^s(\Diamond^s\varphi)) \wedge^s (\Box^s(\Diamond^s(\neg^s\varphi)))) \supset^s \Diamond^s(\Diamond^s\varphi)$  — holds in KT

**abbreviation**(*input*)  $F10 \equiv ((\Box^s(\varphi \supset^s \Box^s\psi)) \wedge^s (\Box^s(\Diamond^s(\neg^s\psi)))) \supset^s \neg^s(\Box^s\psi)$  — holds in KT

**declare** *imp-cong*[*cong del*]

**experiment begin**

**lemma**  $S5: \forall w:W. r R \wedge s R \wedge t R \longrightarrow (\langle W,R,V \rangle, w \models^s F1)$

— nitpick[expect=unknown] — sledgehammer — unkn — proof

**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof

**by** *blast*

**lemma**  $S4: \forall w:W. r R \wedge t R \longrightarrow (\langle W,R,V \rangle, w \models^s F1)$

— nitpick[expect=unknown] — sledgehammer — unkn — proof

**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof

**by** *blast*

**lemma**  $KB4: \forall w:W. s R \wedge t R \longrightarrow (\langle W,R,V \rangle, w \models^s F1)$

— nitpick[expect=unknown] — sledgehammer — unkn — proof

**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof

**by** *blast*

**lemma**  $KTB: \forall w:W. r R \wedge s R \longrightarrow (\langle W,R,V \rangle, w \models^s F1)$

— nitpick[expect=genuine] — sledgehammer — ctex — no prf

**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf

**oops**

**lemma**  $KT: \forall w:W. r R \longrightarrow (\langle W,R,V \rangle, w \models^s F1)$

— nitpick[expect=genuine] — sledgehammer — ctex — no prf

```

apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma KB:  $\forall w:W. s R \longrightarrow (\langle W,R,V \rangle, w \models^s F1)$ 
  — nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma K4:  $\forall w:W. t R \longrightarrow (\langle W,R,V \rangle, w \models^s F1)$ 
  — nitpick[expect=unknown] — sledgehammer — unkn — proof
apply simp — nitpick[expect=unknown] — sledgehammer — unkn — proof
by blast
lemma K:  $\forall w:W. (\langle W,R,V \rangle, w \models^s F1)$ 
  — nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
end

experiment begin
lemma S5:  $\forall w:W. r R \wedge s R \wedge t R \longrightarrow (\langle W,R,V \rangle, w \models^s F2)$ 
  — nitpick[expect=unknown] — sledgehammer — unkn — proof
apply simp — nitpick[expect=unknown] — sledgehammer — unkn — proof
by blast
lemma S4:  $\forall w:W. r R \wedge t R \longrightarrow (\langle W,R,V \rangle, w \models^s F2)$ 
  — nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma KB4:  $\forall w:W. s R \wedge t R \longrightarrow (\langle W,R,V \rangle, w \models^s F2)$ 
  — nitpick[expect=unknown] — sledgehammer — unkn — proof
apply simp — nitpick[expect=unknown] — sledgehammer — unkn — proof
by blast
lemma KTB:  $\forall w:W. r R \wedge s R \longrightarrow (\langle W,R,V \rangle, w \models^s F2)$ 
  — nitpick[expect=unknown] — sledgehammer — unkn — proof
apply simp — nitpick[expect=unknown] — sledgehammer — unkn — proof
by blast
lemma KT:  $\forall w:W. r R \longrightarrow (\langle W,R,V \rangle, w \models^s F2)$ 
  — nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma KB:  $\forall w:W. s R \longrightarrow (\langle W,R,V \rangle, w \models^s F2)$ 
  — nitpick[expect=unknown] — sledgehammer — unkn — proof
apply simp — nitpick[expect=unknown] — sledgehammer — unkn — proof
by blast
lemma K4:  $\forall w:W. t R \longrightarrow (\langle W,R,V \rangle, w \models^s F2)$ 
  — nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma K:  $\forall w:W. (\langle W,R,V \rangle, w \models^s F2)$ 
  — nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops

```

**end**

**experiment begin**

**lemma**  $S5: \forall w:W. r R \wedge s R \wedge t R \longrightarrow (\langle W,R,V \rangle, w \models^s F3)$   
— nitpick[expect=unknown] — sledgehammer — unkn — proof  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**by** *blast*

**lemma**  $S4: \forall w:W. r R \wedge t R \longrightarrow (\langle W,R,V \rangle, w \models^s F3)$   
— nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**oops**

**lemma**  $KB4: \forall w:W. s R \wedge t R \longrightarrow (\langle W,R,V \rangle, w \models^s F3)$   
— nitpick[expect=unknown] — sledgehammer — unkn — proof  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**by** *blast*

**lemma**  $KTB: \forall w:W. r R \wedge s R \longrightarrow (\langle W,R,V \rangle, w \models^s F3)$   
— nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**oops**

**lemma**  $KT: \forall w:W. r R \longrightarrow (\langle W,R,V \rangle, w \models^s F3)$   
— nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**oops**

**lemma**  $KB: \forall w:W. s R \longrightarrow (\langle W,R,V \rangle, w \models^s F3)$   
— nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**oops**

**lemma**  $K4: \forall w:W. t R \longrightarrow (\langle W,R,V \rangle, w \models^s F3)$   
— nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**oops**

**lemma**  $K: \forall w:W. (\langle W,R,V \rangle, w \models^s F3)$   
— nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**oops**

**end**

**experiment begin**

**lemma**  $S5: \forall w:W. r R \wedge s R \wedge t R \longrightarrow (\langle W,R,V \rangle, w \models^s F4)$   
— nitpick[expect=unknown] — sledgehammer — unkn — proof  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**by** *metis*

**lemma**  $S4: \forall w:W. r R \wedge t R \longrightarrow (\langle W,R,V \rangle, w \models^s F4)$   
— nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**by** *metis*

**lemma**  $KB4: \forall w:W. s R \wedge t R \longrightarrow (\langle W,R,V \rangle, w \models^s F4)$   
— nitpick[expect=unknown] — sledgehammer — unkn — proof  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof

by *metis*  
**lemma** *KTB*:  $\forall w:W. \text{r } R \wedge \text{s } R \longrightarrow (\langle W,R,V \rangle, w \models^s F_4)$   
 — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof  
 by *blast*  
**lemma** *KT*:  $\forall w:W. \text{r } R \longrightarrow (\langle W,R,V \rangle, w \models^s F_4)$   
 — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**oops**  
**lemma** *KB*:  $\forall w:W. \text{s } R \longrightarrow (\langle W,R,V \rangle, w \models^s F_4)$   
 — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof  
 by *blast*  
**lemma** *K4*:  $\forall w:W. \text{t } R \longrightarrow (\langle W,R,V \rangle, w \models^s F_4)$   
 — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof  
 by *meson*  
**lemma** *K*:  $\forall w:W. (\langle W,R,V \rangle, w \models^s F_4)$   
 — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**oops**  
**end**

#### experiment begin

**lemma** *S5*:  $\forall w:W. \text{r } R \wedge \text{s } R \wedge \text{t } R \longrightarrow (\langle W,R,V \rangle, w \models^s F_5)$   
 — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof  
 by *blast*  
**lemma** *S4*:  $\forall w:W. \text{r } R \wedge \text{t } R \longrightarrow (\langle W,R,V \rangle, w \models^s F_5)$   
 — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof  
 by *blast*  
**lemma** *KB4*:  $\forall w:W. \text{s } R \wedge \text{t } R \longrightarrow (\langle W,R,V \rangle, w \models^s F_5)$   
 — nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof  
 by *blast*  
**lemma** *KTB*:  $\forall w:W. \text{r } R \wedge \text{s } R \longrightarrow (\langle W,R,V \rangle, w \models^s F_5)$   
 — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**oops**  
**lemma** *KT*:  $\forall w:W. \text{r } R \longrightarrow (\langle W,R,V \rangle, w \models^s F_5)$   
 — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**oops**  
**lemma** *KB*:  $\forall w:W. \text{s } R \longrightarrow (\langle W,R,V \rangle, w \models^s F_5)$   
 — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**oops**  
**lemma** *K4*:  $\forall w:W. \text{t } R \longrightarrow (\langle W,R,V \rangle, w \models^s F_5)$

— nitpick[expect=unknown] — sledgehammer — unkn — no prf  
**apply simp** — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**by blast**  
**lemma K**:  $\forall w:W. (\langle W,R,V \rangle, w \models^s F5)$   
 — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply simp** — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**oops**  
**end**

### experiment begin

**lemma S5**:  $\forall w:W. r R \wedge s R \wedge t R \longrightarrow (\langle W,R,V \rangle, w \models^s F6)$   
 — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**apply simp** — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**by blast**  
**lemma S4**:  $\forall w:W. r R \wedge t R \longrightarrow (\langle W,R,V \rangle, w \models^s F6)$   
 — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply simp** — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**oops**  
**lemma KB4**:  $\forall w:W. s R \wedge t R \longrightarrow (\langle W,R,V \rangle, w \models^s F6)$   
 — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**apply simp** — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**by blast**  
**lemma KTB**:  $\forall w:W. r R \wedge s R \longrightarrow (\langle W,R,V \rangle, w \models^s F6)$   
 — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply simp** — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**oops**  
**lemma KT**:  $\forall w:W. r R \longrightarrow (\langle W,R,V \rangle, w \models^s F6)$   
 — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply simp** — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**oops**  
**lemma KB**:  $\forall w:W. s R \longrightarrow (\langle W,R,V \rangle, w \models^s F6)$   
 — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply simp** — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**oops**  
**lemma K4**:  $\forall w:W. t R \longrightarrow (\langle W,R,V \rangle, w \models^s F6)$   
 — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply simp** — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**oops**  
**lemma K**:  $\forall w:W. (\langle W,R,V \rangle, w \models^s F6)$   
 — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply simp** — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**oops**  
**end**

### experiment begin

**lemma S5**:  $\forall w:W. r R \wedge s R \wedge t R \longrightarrow (\langle W,R,V \rangle, w \models^s F7)$   
 — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**apply simp** — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**by blast**

**lemma**  $S_4$ :  $\forall w:W. r R \wedge t R \longrightarrow (\langle W,R,V \rangle, w \models^s F7)$   
 — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**oops**  
**lemma**  $KB_4$ :  $\forall w:W. s R \wedge t R \longrightarrow (\langle W,R,V \rangle, w \models^s F7)$   
 — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**by** *blast*  
**lemma**  $KTB$ :  $\forall w:W. r R \wedge s R \longrightarrow (\langle W,R,V \rangle, w \models^s F7)$   
 — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**oops**  
**lemma**  $KT$ :  $\forall w:W. r R \longrightarrow (\langle W,R,V \rangle, w \models^s F7)$   
 — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**oops**  
**lemma**  $KB$ :  $\forall w:W. s R \longrightarrow (\langle W,R,V \rangle, w \models^s F7)$   
 — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**oops**  
**lemma**  $K_4$ :  $\forall w:W. t R \longrightarrow (\langle W,R,V \rangle, w \models^s F7)$   
 — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**oops**  
**lemma**  $K$ :  $\forall w:W. (\langle W,R,V \rangle, w \models^s F7)$   
 — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**oops**  
**end**

### experiment begin

**lemma**  $S5$ :  $\forall w:W. r R \wedge s R \wedge t R \longrightarrow (\langle W,R,V \rangle, w \models^s F8)$   
 — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**by** *blast*  
**lemma**  $S_4$ :  $\forall w:W. r R \wedge t R \longrightarrow (\langle W,R,V \rangle, w \models^s F8)$   
 — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**by** *blast*  
**lemma**  $KB_4$ :  $\forall w:W. s R \wedge t R \longrightarrow (\langle W,R,V \rangle, w \models^s F8)$   
 — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**by** *blast*  
**lemma**  $KTB$ :  $\forall w:W. r R \wedge s R \longrightarrow (\langle W,R,V \rangle, w \models^s F8)$   
 — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**by** *blast*  
**lemma**  $KT$ :  $\forall w:W. r R \longrightarrow (\langle W,R,V \rangle, w \models^s F8)$   
 — nitpick[expect=unknown] — sledgehammer — unkn — proof

```

apply simp — nitpick[expect=unknown] — sledgehammer — unkn — proof
by blast
lemma KB:  $\forall w:W. s R \longrightarrow (\langle W,R,V \rangle, w \models^s F8)$ 
  — nitpick[expect=unknown] — sledgehammer — unkn — proof
apply simp — nitpick[expect=unknown] — sledgehammer — unkn — proof
by blast
lemma K4:  $\forall w:W. t R \longrightarrow (\langle W,R,V \rangle, w \models^s F8)$ 
  — nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma K:  $\forall w:W. (\langle W,R,V \rangle, w \models^s F8)$ 
  — nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
end

experiment begin
lemma S5:  $\forall w:W. r R \wedge s R \wedge t R \longrightarrow (\langle W,R,V \rangle, w \models^s F9)$ 
  — nitpick[expect=unknown] — sledgehammer — unkn — proof
apply simp — nitpick[expect=unknown] — sledgehammer — unkn — proof
by blast
lemma S4:  $\forall w:W. r R \wedge t R \longrightarrow (\langle W,R,V \rangle, w \models^s F9)$ 
  — nitpick[expect=unknown] — sledgehammer — unkn — proof
apply simp — nitpick[expect=unknown] — sledgehammer — unkn — proof
by blast
lemma KB4:  $\forall w:W. s R \wedge t R \longrightarrow (\langle W,R,V \rangle, w \models^s F9)$ 
  — nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma KTB:  $\forall w:W. r R \wedge s R \longrightarrow (\langle W,R,V \rangle, w \models^s F9)$ 
  — nitpick[expect=unknown] — sledgehammer — unkn — proof
apply simp — nitpick[expect=unknown] — sledgehammer — unkn — proof
by blast
lemma KT:  $\forall w:W. r R \longrightarrow (\langle W,R,V \rangle, w \models^s F9)$ 
  — nitpick[expect=unknown] — sledgehammer — unkn — proof
apply simp — nitpick[expect=unknown] — sledgehammer — unkn — proof
by blast
lemma KB:  $\forall w:W. s R \longrightarrow (\langle W,R,V \rangle, w \models^s F9)$ 
  — nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma K4:  $\forall w:W. t R \longrightarrow (\langle W,R,V \rangle, w \models^s F9)$ 
  — nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma K:  $\forall w:W. (\langle W,R,V \rangle, w \models^s F9)$ 
  — nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops

```

**end**

**experiment begin**

**lemma** *S5*:  $\forall w:W. r R \wedge s R \wedge t R \longrightarrow (\langle W,R,V \rangle, w \models^s F10)$

— nitpick[expect=unknown] — sledgehammer — unkn — no prf

**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**by** *blast*

**lemma** *S4*:  $\forall w:W. r R \wedge t R \longrightarrow (\langle W,R,V \rangle, w \models^s F10)$

— nitpick[expect=unknown] — sledgehammer — unkn — no prf

**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**by** *blast*

**lemma** *KB4*:  $\forall w:W. s R \wedge t R \longrightarrow (\langle W,R,V \rangle, w \models^s F10)$

— nitpick[expect=genuine] — sledgehammer — ctex — no prf

**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**oops**

**lemma** *KTB*:  $\forall w:W. r R \wedge s R \longrightarrow (\langle W,R,V \rangle, w \models^s F10)$

— nitpick[expect=unknown] — sledgehammer — unkn — no prf

**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**by** *blast*

**lemma** *KT*:  $\forall w:W. r R \longrightarrow (\langle W,R,V \rangle, w \models^s F10)$

— nitpick[expect=unknown] — sledgehammer — unkn — no prf

**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**by** *blast*

**lemma** *KB*:  $\forall w:W. s R \longrightarrow (\langle W,R,V \rangle, w \models^s F10)$

— nitpick[expect=genuine] — sledgehammer — ctex — no prf

**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**oops**

**lemma** *K4*:  $\forall w:W. t R \longrightarrow (\langle W,R,V \rangle, w \models^s F10)$

— nitpick[expect=genuine] — sledgehammer — ctex — no prf

**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**oops**

**lemma** *K*:  $\forall w:W. (\langle W,R,V \rangle, w \models^s F10)$

— nitpick[expect=genuine] — sledgehammer — ctex — no prf

**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**oops**

**end**

— Summary of experiments: Nitpick: ctex=84 (with simp 42, without simp 42), none=0, unknwn=76 (with simp 38, without simp 38) Sledgehammer: proof=66 (with simp 38, without simp 28), no prf=94 (with simp 42, without simp 52)

— No conflicts

**end**

## 8.5 Tests with the (minimal) shallow embedding: axiom schemata

**theory** *PMLinHOL-shallow-minimal-further-tests-1*

**imports** *PMLinHOL-shallow-minimal* — C.B., 2025

**begin**

— What is the weakest modal logic in which the following test formulas F1-F10 are provable?

— Test with schematic axioms

**abbreviation**(*input*)  $AxT \equiv \forall \varphi. \models^m (\Box^m \varphi) \supset^m \varphi$

**abbreviation**(*input*)  $AxB \equiv \forall \varphi. \models^m \varphi \supset^m \Box^m (\Diamond^m \varphi)$

**abbreviation**(*input*)  $Ax4 \equiv \forall \varphi. \models^m (\Box^m \varphi) \supset^m \Box^m (\Box^m \varphi)$

**consts**  $\varphi::\sigma \ \psi::\sigma$

**abbreviation**(*input*)  $F1 \equiv (\Diamond^m (\Diamond^m \varphi)) \supset^m \Diamond^m \varphi$  — holds in K4

**abbreviation**(*input*)  $F2 \equiv (\Diamond^m (\Box^m \varphi)) \supset^m \Box^m (\Diamond^m \varphi)$  — holds in KB

**abbreviation**(*input*)  $F3 \equiv (\Diamond^m (\Box^m \varphi)) \supset^m \Box^m \varphi$  — holds in KB4

**abbreviation**(*input*)  $F4 \equiv (\Box^m (\Diamond^m (\Box^m (\Diamond^m \varphi)))) \supset^m \Box^m (\Diamond^m \varphi)$  — holds in KB/K4

**abbreviation**(*input*)  $F5 \equiv (\Diamond^m (\varphi \wedge^m (\Diamond^m \psi))) \supset^m ((\Diamond^m \varphi) \wedge^m (\Diamond^m \psi))$  — holds in K4

**abbreviation**(*input*)  $F6 \equiv ((\Box^m (\varphi \supset^m \psi)) \wedge^m (\Diamond^m (\Box^m (\neg^m \psi)))) \supset^m \neg^m (\Diamond^m \psi)$  — holds in KB4

**abbreviation**(*input*)  $F7 \equiv (\Diamond^m \varphi) \supset^m (\Box^m (\varphi \vee^m \Diamond^m \varphi))$  — holds in KB4

**abbreviation**(*input*)  $F8 \equiv (\Diamond^m (\Box^m \varphi)) \supset^m (\varphi \vee^m \Diamond^m \varphi)$  — holds in KT/KB

**abbreviation**(*input*)  $F9 \equiv ((\Box^m (\Diamond^m \varphi)) \wedge^m (\Box^m (\Diamond^m (\neg^m \varphi)))) \supset^m \Diamond^m (\Diamond^m \varphi)$  — holds in KT

**abbreviation**(*input*)  $F10 \equiv ((\Box^m (\varphi \supset^m \Box^m \psi)) \wedge^m (\Box^m (\Diamond^m (\neg^m \psi)))) \supset^m \neg^m (\Box^m \psi)$  — holds in KT

**declare** *imp-cong*[*cong del*]

**nitpick-params**

**experiment begin**

**lemma**  $S5: AxT \wedge AxB \wedge Ax4 \longrightarrow \models^m F1$

— nitpick[expect=none] — sledgehammer — none — proof

**apply** *simp* — nitpick[expect=none] — sledgehammer — none — proof

**by** *metis*

**lemma**  $S4: AxT \wedge Ax4 \longrightarrow \models^m F1$

— nitpick[expect=none] — sledgehammer — none — proof

**apply** *simp* — nitpick[expect=none] — sledgehammer — none — proof

**by** *metis*

**lemma**  $KB4: AxB \wedge Ax4 \longrightarrow \models^m F1$

— nitpick[expect=none] — sledgehammer — none — proof

**apply** *simp* — nitpick[expect=none] — sledgehammer — none — proof

**by** *metis*

**lemma**  $KTB: AxT \wedge AxB \longrightarrow \models^m F1$

— nitpick[expect=genuine] — sledgehammer — ctx — no prf

**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctx — no prf

**oops**

**lemma**  $KT: AxT \longrightarrow \models^m F1$

— nitpick[expect=genuine] — sledgehammer — ctx — no prf

**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctx — no prf

**oops**

**lemma**  $KB: AxB \longrightarrow \models^m F1$

```

— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma K4:  $Ax4 \longrightarrow \models^m F1$ 
— nitpick[expect=none] — sledgehammer — none — proof
apply simp — nitpick[expect=none] — sledgehammer — none — proof
by metis
lemma K:  $\models^m F1$ 
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
end

experiment begin
lemma S5:  $AxT \wedge AxB \wedge Ax4 \longrightarrow \models^m F2$ 
— nitpick[expect=none] — sledgehammer — none — proof
apply simp — nitpick[expect=none] — sledgehammer — none — proof
by (metis NegM-def)
lemma S4:  $AxT \wedge Ax4 \longrightarrow \models^m F2$ 
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma KB4:  $AxB \wedge Ax4 \longrightarrow \models^m F2$ 
— nitpick[expect=none] — sledgehammer — none — proof
apply simp — nitpick[expect=none] — sledgehammer — none — proof
by (metis NegM-def)
lemma KTB:  $AxT \wedge AxB \longrightarrow \models^m F2$ 
— nitpick[expect=none] — sledgehammer — none — proof
apply simp — nitpick[expect=none] — sledgehammer — none — proof
by (metis NegM-def)
lemma KT:  $AxT \longrightarrow \models^m F2$ 
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma KB:  $AxB \longrightarrow \models^m F2$ 
— nitpick[expect=none] — sledgehammer — none — proof
apply simp — nitpick[expect=none] — sledgehammer — none — proof
by (metis NegM-def)
lemma K4:  $Ax4 \longrightarrow \models^m F2$ 
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma K:  $\models^m F2$ 
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
end

experiment begin

```

**lemma** *S5*:  $AxT \wedge AxB \wedge Ax4 \longrightarrow \models^m F3$   
 — nitpick[expect=none] — sledgehammer — none — proof  
**apply** *simp* — nitpick[expect=none] — sledgehammer — none — proof  
**by** (*metis NegM-def*)  
**lemma** *S4*:  $AxT \wedge Ax4 \longrightarrow \models^m F3$   
 — nitpick[expect=genuine] — sledgehammer — ctext — no prf  
**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctext — no prf  
**oops**  
**lemma** *KB4*:  $AxB \wedge Ax4 \longrightarrow \models^m F3$   
 — nitpick[expect=none] — sledgehammer — none — proof  
**apply** *simp* — nitpick[expect=none] — sledgehammer — none — proof  
**by** (*metis NegM-def*)  
**lemma** *KTB*:  $AxT \wedge AxB \longrightarrow \models^m F3$   
 — nitpick[expect=genuine] — sledgehammer — ctext — no prf  
**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctext — no prf  
**oops**  
**lemma** *KT*:  $AxT \longrightarrow \models^m F3$   
 — nitpick[expect=genuine] — sledgehammer — ctext — no prf  
**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctext — no prf  
**oops**  
**lemma** *KB*:  $AxB \longrightarrow \models^m F3$   
 — nitpick[expect=genuine] — sledgehammer — ctext — no prf  
**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctext — no prf  
**oops**  
**lemma** *K4*:  $Ax4 \longrightarrow \models^m F3$   
 — nitpick[expect=genuine] — sledgehammer — ctext — no prf  
**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctext — no prf  
**oops**  
**lemma** *K*:  $\models^m F3$   
 — nitpick[expect=genuine] — sledgehammer — ctext — no prf  
**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctext — no prf  
**oops**  
**end**

#### experiment begin

**lemma** *S5*:  $AxT \wedge AxB \wedge Ax4 \longrightarrow \models^m F4$   
 — nitpick[expect=none] — sledgehammer — none — proof  
**apply** *simp* — nitpick[expect=none] — sledgehammer — none — proof  
**by** (*metis NegM-def*)  
**lemma** *S4*:  $AxT \wedge Ax4 \longrightarrow \models^m F4$   
 — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**apply** *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof  
**by** *smt*  
**lemma** *KB4*:  $AxB \wedge Ax4 \longrightarrow \models^m F4$   
 — nitpick[expect=none] — sledgehammer — none — proof  
**apply** *simp* — nitpick[expect=none] — sledgehammer — none — proof  
**by** (*metis NegM-def*)  
**lemma** *KTB*:  $AxT \wedge AxB \longrightarrow \models^m F4$   
 — nitpick[expect=none] — sledgehammer — none — proof

```

apply simp — nitpick[expect=none] — sledgehammer — none — proof
by (metis NegM-def)
lemma KT:  $AxT \longrightarrow \models^m F4$ 
  — nitpick[expect=genuine] — sledgehammer — ctext — no prf
apply simp — nitpick[expect=genuine] — sledgehammer — ctext — no prf
oops
lemma KB:  $AxB \longrightarrow \models^m F4$ 
  — nitpick[expect=none] — sledgehammer — none — proof
apply simp — nitpick[expect=none] — sledgehammer — none — proof
by (metis NegM-def)
lemma K4:  $Ax4 \longrightarrow \models^m F4$ 
  — nitpick[expect=unknown] — sledgehammer — unkn — proof
apply simp — nitpick[expect=unknown] — sledgehammer — unkn — proof
by smt
lemma K:  $\models^m F4$ 
  — nitpick[expect=genuine] — sledgehammer — ctext — no prf
apply simp — nitpick[expect=genuine] — sledgehammer — ctext — no prf
oops
end

experiment begin
lemma S5:  $AxT \wedge AxB \wedge Ax4 \longrightarrow \models^m F5$ 
  — nitpick[expect=none] — sledgehammer — none — proof
apply simp — nitpick[expect=none] — sledgehammer — none — proof
by metis
lemma S4:  $AxT \wedge Ax4 \longrightarrow \models^m F5$ 
  — nitpick[expect=none] — sledgehammer — none — proof
apply simp — nitpick[expect=none] — sledgehammer — none — proof
by metis
lemma KB4:  $AxB \wedge Ax4 \longrightarrow \models^m F5$ 
  — nitpick[expect=none] — sledgehammer — none — proof
apply simp — nitpick[expect=none] — sledgehammer — none — proof
by metis
lemma KTB:  $AxT \wedge AxB \longrightarrow \models^m F5$ 
  — nitpick[expect=genuine] — sledgehammer — ctext — no prf
apply simp — nitpick[expect=genuine] — sledgehammer — ctext — no prf
oops
lemma KT:  $AxT \longrightarrow \models^m F5$ 
  — nitpick[expect=genuine] — sledgehammer — ctext — no prf
apply simp — nitpick[expect=genuine] — sledgehammer — ctext — no prf
oops
lemma KB:  $AxB \longrightarrow \models^m F5$ 
  — nitpick[expect=genuine] — sledgehammer — ctext — no prf
apply simp — nitpick[expect=genuine] — sledgehammer — ctext — no prf
oops
lemma K4:  $Ax4 \longrightarrow \models^m F5$ 
  — nitpick[expect=none] — sledgehammer — none — proof
apply simp — nitpick[expect=none] — sledgehammer — none — proof
by metis

```

```

lemma K:  $\models^m F5$ 
  — nitpick[expect=genuine] — sledgehammer — ctext — no prf
  apply simp — nitpick[expect=genuine] — sledgehammer — ctext — no prf
  oops
end

```

**experiment begin**

```

lemma S5:  $AxT \wedge AxB \wedge Ax4 \longrightarrow \models^m F6$ 
  — nitpick[expect=none] — sledgehammer — none — proof
  apply simp — nitpick[expect=none] — sledgehammer — none — proof
  by (metis NegM-def)
lemma S4:  $AxT \wedge Ax4 \longrightarrow \models^m F6$ 
  — nitpick[expect=genuine] — sledgehammer — ctext — no prf
  apply simp — nitpick[expect=genuine] — sledgehammer — ctext — no prf
  oops
lemma KB4:  $AxB \wedge Ax4 \longrightarrow \models^m F6$ 
  — nitpick[expect=none] — sledgehammer — none — proof
  apply simp — nitpick[expect=none] — sledgehammer — none — proof
  by (metis NegM-def)
lemma KTB:  $AxT \wedge AxB \longrightarrow \models^m F6$ 
  — nitpick[expect=genuine] — sledgehammer — ctext — no prf
  apply simp — nitpick[expect=genuine] — sledgehammer — ctext — no prf
  oops
lemma KT:  $AxT \longrightarrow \models^m F6$ 
  — nitpick[expect=genuine] — sledgehammer — ctext — no prf
  apply simp — nitpick[expect=genuine] — sledgehammer — ctext — no prf
  oops
lemma KB:  $AxB \longrightarrow \models^m F6$ 
  — nitpick[expect=genuine] — sledgehammer — ctext — no prf
  apply simp — nitpick[expect=genuine] — sledgehammer — ctext — no prf
  oops
lemma K4:  $Ax4 \longrightarrow \models^m F6$ 
  — nitpick[expect=genuine] — sledgehammer — ctext — no prf
  apply simp — nitpick[expect=genuine] — sledgehammer — ctext — no prf
  oops
lemma K:  $\models^m F6$ 
  — nitpick[expect=genuine] — sledgehammer — ctext — no prf
  apply simp — nitpick[expect=genuine] — sledgehammer — ctext — no prf
  oops
end

```

**experiment begin**

```

lemma S5:  $AxT \wedge AxB \wedge Ax4 \longrightarrow \models^m F7$ 
  — nitpick[expect=none] — sledgehammer — none — proof
  apply simp — nitpick[expect=none] — sledgehammer — none — proof
  by (metis NegM-def)
lemma S4:  $AxT \wedge Ax4 \longrightarrow \models^m F7$ 
  — nitpick[expect=genuine] — sledgehammer — ctext — no prf
  apply simp — nitpick[expect=genuine] — sledgehammer — ctext — no prf

```

```

oops
lemma  $KB_4: AxB \wedge Ax_4 \longrightarrow \models^m F7$ 
  — nitpick[expect=none] — sledgehammer — none — proof
  apply simp — nitpick[expect=none] — sledgehammer — none — proof
  by (metis NegM-def)
lemma  $KTB: AxT \wedge AxB \longrightarrow \models^m F7$ 
  — nitpick[expect=genuine] — sledgehammer — ctext — no prf
  apply simp — nitpick[expect=genuine] — sledgehammer — ctext — no prf
oops
lemma  $KT: AxT \longrightarrow \models^m F7$ 
  — nitpick[expect=genuine] — sledgehammer — ctext — no prf
  apply simp — nitpick[expect=genuine] — sledgehammer — ctext — no prf
oops
lemma  $KB: AxB \longrightarrow \models^m F7$ 
  — nitpick[expect=genuine] — sledgehammer — ctext — no prf
  apply simp — nitpick[expect=genuine] — sledgehammer — ctext — no prf
oops
lemma  $K_4: Ax_4 \longrightarrow \models^m F7$ 
  — nitpick[expect=genuine] — sledgehammer — ctext — no prf
  apply simp — nitpick[expect=genuine] — sledgehammer — ctext — no prf
oops
lemma  $K: \models^m F7$ 
  — nitpick[expect=genuine] — sledgehammer — ctext — no prf
  apply simp — nitpick[expect=genuine] — sledgehammer — ctext — no prf
oops
end

experiment begin
lemma  $S5: AxT \wedge AxB \wedge Ax_4 \longrightarrow \models^m F8$ 
  — nitpick[expect=none] — sledgehammer — none — proof
  apply simp — nitpick[expect=none] — sledgehammer — none — proof
  by metis
lemma  $S_4: AxT \wedge Ax_4 \longrightarrow \models^m F8$ 
  — nitpick[expect=none] — sledgehammer — none — proof
  apply simp — nitpick[expect=none] — sledgehammer — none — proof
  by metis
lemma  $KB_4: AxB \wedge Ax_4 \longrightarrow \models^m F8$ 
  — nitpick[expect=none] — sledgehammer — none — proof
  apply simp — nitpick[expect=none] — sledgehammer — none — proof
  by (metis NegM-def)
lemma  $KTB: AxT \wedge AxB \longrightarrow \models^m F8$ 
  — nitpick[expect=none] — sledgehammer — none — proof
  apply simp — nitpick[expect=none] — sledgehammer — none — proof
  by metis
lemma  $KT: AxT \longrightarrow \models^m F8$ 
  — nitpick[expect=none] — sledgehammer — none — proof
  apply simp — nitpick[expect=none] — sledgehammer — none — proof
  by metis
lemma  $KB: AxB \longrightarrow \models^m F8$ 

```

```

— nitpick[expect=none] — sledgehammer — none — proof
apply simp — nitpick[expect=none] — sledgehammer — none — proof
by (metis NegM-def)
lemma K4:  $Ax4 \longrightarrow \models^m F8$ 
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma K:  $\models^m F8$ 
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
end

```

**experiment begin**

```

lemma S5:  $AxT \wedge AxB \wedge Ax4 \longrightarrow \models^m F9$ 
— nitpick[expect=none] — sledgehammer — none — proof
apply simp — nitpick[expect=none] — sledgehammer — none — proof
by metis
lemma S4:  $AxT \wedge Ax4 \longrightarrow \models^m F9$ 
— nitpick[expect=none] — sledgehammer — none — proof
apply simp — nitpick[expect=none] — sledgehammer — none — proof
by metis
lemma KB4:  $AxB \wedge Ax4 \longrightarrow \models^m F9$ 
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma KTB:  $AxT \wedge AxB \longrightarrow \models^m F9$ 
— nitpick[expect=none] — sledgehammer — none — proof
apply simp — nitpick[expect=none] — sledgehammer — none — proof
by metis
lemma KT:  $AxT \longrightarrow \models^m F9$ 
— nitpick[expect=none] — sledgehammer — none — proof
apply simp — nitpick[expect=none] — sledgehammer — none — proof
by metis
lemma KB:  $AxB \longrightarrow \models^m F9$ 
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma K4:  $Ax4 \longrightarrow \models^m F9$ 
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma K:  $\models^m F9$ 
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
end

```

**experiment begin**

```

lemma S5:  $AxT \wedge AxB \wedge Ax4 \longrightarrow \models^m F10$ 
  — nitpick[expect=none] — sledgehammer — none — no prf
  apply simp — nitpick[expect=none] — sledgehammer — none — proof
  by metis
lemma S4:  $AxT \wedge Ax4 \longrightarrow \models^m F10$ 
  — nitpick[expect=none] — sledgehammer — none — no prf
  apply simp — nitpick[expect=none] — sledgehammer — none — proof
  by metis
lemma KB4:  $AxB \wedge Ax4 \longrightarrow \models^m F10$ 
  — nitpick[expect=genuine] — sledgehammer — ctext — no prf
  apply simp — nitpick[expect=genuine] — sledgehammer — ctext — no prf
  oops
lemma KTB:  $AxT \wedge AxB \longrightarrow \models^m F10$ 
  — nitpick[expect=none] — sledgehammer — none — no prf
  apply simp — nitpick[expect=none] — sledgehammer — none — proof
  by metis
lemma KT:  $AxT \longrightarrow \models^m F10$ 
  — nitpick[expect=none] — sledgehammer — none — proof
  apply simp — nitpick[expect=none] — sledgehammer — none — proof
  by metis
lemma KB:  $AxB \longrightarrow \models^m F10$ 
  — nitpick[expect=genuine] — sledgehammer — ctext — no prf
  apply simp — nitpick[expect=genuine] — sledgehammer — ctext — no prf
  oops
lemma K4:  $Ax4 \longrightarrow \models^m F10$ 
  — nitpick[expect=genuine] — sledgehammer — ctext — no prf
  apply simp — nitpick[expect=genuine] — sledgehammer — ctext — no prf
  oops
lemma K:  $\models^m F10$ 
  — nitpick[expect=genuine] — sledgehammer — ctext — no prf
  apply simp — nitpick[expect=genuine] — sledgehammer — ctext — no prf
  oops
end

```

— Summary of experiments: Nitpick: ctext=84 (with simp 42, without simp 42), none=72 (with simp 36, without simp 36), unknown=4 (with simp 2, without simp 2) Sledgehammer: proof=73 (with simp 38, without simp 35), no prf=87 (with simp 42, without simp 45)

— No conflicts  
**end**

## 8.6 Tests with the (minimal) shallow embedding: semantic frame conditions

```

theory PMLinHOL-shallow-minimal-further-tests-2
  imports PMLinHOL-shallow-minimal
begin

```

— What is the weakest modal logic in which the following test formulas F1-F10

are provable?

— Test with semantic conditions

**abbreviation**(*input*) *refl* (r) **where**  $r \equiv \lambda R. \text{ reflexive } R$

**abbreviation**(*input*) *sym* (s) **where**  $s \equiv \lambda R. \text{ symmetric } R$

**abbreviation**(*input*) *tra* (t) **where**  $t \equiv \lambda R. \text{ transitive } R$

**consts**  $\varphi::\sigma \ \psi::\sigma$

**abbreviation**(*input*)  $F1 \equiv (\diamond^m(\diamond^m\varphi)) \supset^m \diamond^m\varphi$  — holds in K4

**abbreviation**(*input*)  $F2 \equiv (\diamond^m(\Box^m\varphi)) \supset^m \Box^m(\diamond^m\varphi)$  — holds in KB

**abbreviation**(*input*)  $F3 \equiv (\diamond^m(\Box^m\varphi)) \supset^m \Box^m\varphi$  — holds in KB4

**abbreviation**(*input*)  $F4 \equiv (\Box^m(\diamond^m(\Box^m(\diamond^m\varphi)))) \supset^m \Box^m(\diamond^m\varphi)$  — holds in KB

**abbreviation**(*input*)  $F5 \equiv (\diamond^m(\varphi \wedge^m (\diamond^m\psi))) \supset^m ((\diamond^m\varphi) \wedge^m (\diamond^m\psi))$  — holds in K4

**abbreviation**(*input*)  $F6 \equiv ((\Box^m(\varphi \supset^m \psi)) \wedge^m (\Box^m(\Box^m(\neg^m\psi)))) \supset^m \neg^m(\diamond^m\psi)$   
— holds in KB4

**abbreviation**(*input*)  $F7 \equiv (\diamond^m\varphi) \supset^m (\Box^m(\varphi \vee^m \diamond^m\varphi))$  — holds in KB4

**abbreviation**(*input*)  $F8 \equiv (\diamond^m(\Box^m\varphi)) \supset^m (\varphi \vee^m \diamond^m\varphi)$  — holds in KT and in KB

**abbreviation**(*input*)  $F9 \equiv ((\Box^m(\diamond^m\varphi)) \wedge^m (\Box^m(\Box^m(\neg^m\varphi)))) \supset^m \diamond^m(\diamond^m\varphi)$   
— holds in KT

**abbreviation**(*input*)  $F10 \equiv ((\Box^m(\varphi \supset^m \Box^m\psi)) \wedge^m (\Box^m(\Box^m(\neg^m\psi)))) \supset^m \neg^m(\Box^m\psi)$   
— holds in KT

**declare** *imp-cong*[*cong del*]

**experiment begin**

**lemma**  $S5: r \ R \wedge s \ R \wedge t \ R \longrightarrow (\models^m F1)$

— nitpick[expect=none] — sledgehammer — none — proof

**apply** *simp* — nitpick[expect=none] — sledgehammer — none — proof

**by** *metis*

**lemma**  $S4: r \ R \wedge t \ R \longrightarrow (\models^m F1)$

— nitpick[expect=none] — sledgehammer — none — proof

**apply** *simp* — nitpick[expect=none] — sledgehammer — none — proof

**by** *metis*

**lemma**  $KB4: s \ R \wedge t \ R \longrightarrow (\models^m F1)$

— nitpick[expect=none] — sledgehammer — none — proof

**apply** *simp* — nitpick[expect=none] — sledgehammer — none — proof

**by** *metis*

**lemma**  $KTB: r \ R \wedge s \ R \longrightarrow (\models^m F1)$

— nitpick[expect=genuine] — sledgehammer — ctex — no prf

**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf

**oops**

**lemma**  $KT: r \ R \longrightarrow (\models^m F1)$

— nitpick[expect=genuine] — sledgehammer — ctex — no prf

**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf

**oops**

**lemma**  $KB: s \ R \longrightarrow (\models^m F1)$

— nitpick[expect=genuine] — sledgehammer — ctex — no prf

**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf

```

oops
lemma  $K4$ :  $t R \longrightarrow (\models^m F1)$ 
  — nitpick[expect=none] — sledgehammer — none — proof
  apply simp — nitpick[expect=none] — sledgehammer — none — proof
  by metis
lemma  $K$ :  $(\models^m F1)$ 
  — nitpick[expect=genuine] — sledgehammer — ctex — no prf
  apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
end

```

**experiment begin**

```

lemma  $S5$ :  $r R \wedge s R \wedge t R \longrightarrow (\models^m F2)$ 
  — nitpick[expect=none] — sledgehammer — none — proof
  apply simp — nitpick[expect=none] — sledgehammer — none — proof
  by metis
lemma  $S4$ :  $r R \wedge t R \longrightarrow (\models^m F2)$ 
  — nitpick[expect=genuine] — sledgehammer — ctex — no prf
  apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma  $KB4$ :  $s R \wedge t R \longrightarrow (\models^m F2)$ 
  — nitpick[expect=none] — sledgehammer — none — proof
  apply simp — nitpick[expect=none] — sledgehammer — none — proof
  by metis
lemma  $KTB$ :  $r R \wedge s R \longrightarrow (\models^m F2)$ 
  — nitpick[expect=none] — sledgehammer — none — proof
  apply simp — nitpick[expect=none] — sledgehammer — none — proof
  by metis
lemma  $KT$ :  $r R \longrightarrow (\models^m F2)$ 
  — nitpick[expect=genuine] — sledgehammer — ctex — no prf
  apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma  $KB$ :  $s R \longrightarrow (\models^m F2)$ 
  — nitpick[expect=none] — sledgehammer — none — proof
  apply simp — nitpick[expect=none] — sledgehammer — none — proof
  by metis
lemma  $K4$ :  $t R \longrightarrow (\models^m F2)$ 
  — nitpick[expect=genuine] — sledgehammer — ctex — no prf
  apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma  $K$ :  $(\models^m F2)$ 
  — nitpick[expect=genuine] — sledgehammer — ctex — no prf
  apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
end

```

**experiment begin**

```

lemma  $S5$ :  $r R \wedge s R \wedge t R \longrightarrow (\models^m F3)$ 
  — nitpick[expect=none] — sledgehammer — none — proof

```

```

apply simp — nitpick[expect=none] — sledgehammer — none — proof
by metis
lemma S4: r R ∧ t R → (|=m F3)
  — nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma KB4: s R ∧ t R → (|=m F3)
  — nitpick[expect=none] — sledgehammer — none — proof
apply simp — nitpick[expect=none] — sledgehammer — none — proof
by metis
lemma KTB: r R ∧ s R → (|=m F3)
  — nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma KT: r R → (|=m F3)
  — nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma KB: s R → (|=m F3)
  — nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma K4: t R → (|=m F3)
  — nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma K: (|=m F3)
  — nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
end

```

### experiment begin

```

lemma S5: r R ∧ s R ∧ t R → (|=m F4)
  — nitpick[expect=none] — sledgehammer — none — proof
apply simp — nitpick[expect=none] — sledgehammer — none — proof
by metis
lemma S4: r R ∧ t R → (|=m F4)
  — nitpick[expect=none] — sledgehammer — none — proof
apply simp — nitpick[expect=none] — sledgehammer — none — proof
by metis
lemma KB4: s R ∧ t R → (|=m F4)
  — nitpick[expect=none] — sledgehammer — none — proof
apply simp — nitpick[expect=none] — sledgehammer — none — proof
by metis
lemma KTB: r R ∧ s R → (|=m F4)
  — nitpick[expect=none] — sledgehammer — none — proof
apply simp — nitpick[expect=none] — sledgehammer — none — proof
by metis

```

**lemma** *KT*:  $r R \longrightarrow (\models^m F4)$   
 — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**oops**

**lemma** *KB*:  $s R \longrightarrow (\models^m F4)$   
 — nitpick[expect=none] — sledgehammer — none — proof  
**apply** *simp* — nitpick[expect=none] — sledgehammer — none — proof  
**by** *metis*

**lemma** *K4*:  $t R \longrightarrow (\models^m F4)$   
 — nitpick[expect=none] — sledgehammer — none — proof  
**apply** *simp* — nitpick[expect=none] — sledgehammer — none — proof  
**by** *metis*

**lemma** *K*:  $(\models^m F4)$   
 — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**oops**  
**end**

**experiment begin**

**lemma** *S5*:  $r R \wedge s R \wedge t R \longrightarrow (\models^m F5)$   
 — nitpick[expect=none] — sledgehammer — none — proof  
**apply** *simp* — nitpick[expect=none] — sledgehammer — none — proof  
**by** *metis*

**lemma** *S4*:  $r R \wedge t R \longrightarrow (\models^m F5)$   
 — nitpick[expect=none] — sledgehammer — none — proof  
**apply** *simp* — nitpick[expect=none] — sledgehammer — none — proof  
**by** *metis*

**lemma** *KB4*:  $s R \wedge t R \longrightarrow (\models^m F5)$   
 — nitpick[expect=none] — sledgehammer — none — proof  
**apply** *simp* — nitpick[expect=none] — sledgehammer — none — proof  
**by** *metis*

**lemma** *KTB*:  $r R \wedge s R \longrightarrow (\models^m F5)$   
 — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**oops**

**lemma** *KT*:  $r R \longrightarrow (\models^m F5)$   
 — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**oops**

**lemma** *KB*:  $s R \longrightarrow (\models^m F5)$   
 — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**oops**

**lemma** *K4*:  $t R \longrightarrow (\models^m F5)$   
 — nitpick[expect=none] — sledgehammer — none — proof  
**apply** *simp* — nitpick[expect=none] — sledgehammer — none — proof  
**by** *metis*

**lemma** *K*:  $(\models^m F5)$   
 — nitpick[expect=genuine] — sledgehammer — ctex — no prf

```

apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
end

```

**experiment begin**

```

lemma S5: r R ∧ s R ∧ t R → (|=m F6)
  — nitpick[expect=none] — sledgehammer — none — proof
  apply simp — nitpick[expect=none] — sledgehammer — none — proof
  by metis
lemma S4: r R ∧ t R → (|=m F6)
  — nitpick[expect=genuine] — sledgehammer — ctex — no prf
  apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf
  oops
lemma KB4: s R ∧ t R → (|=m F6)
  — nitpick[expect=none] — sledgehammer — none — proof
  apply simp — nitpick[expect=none] — sledgehammer — none — proof
  by metis
lemma KTB: r R ∧ s R → (|=m F6)
  — nitpick[expect=genuine] — sledgehammer — ctex — no prf
  apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf
  oops
lemma KT: r R → (|=m F6)
  — nitpick[expect=genuine] — sledgehammer — ctex — no prf
  apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf
  oops
lemma KB: s R → (|=m F6)
  — nitpick[expect=genuine] — sledgehammer — ctex — no prf
  apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf
  oops
lemma K4: t R → (|=m F6)
  — nitpick[expect=genuine] — sledgehammer — ctex — no prf
  apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf
  oops
lemma K: (|=m F6)
  — nitpick[expect=genuine] — sledgehammer — ctex — no prf
  apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf
  oops
end

```

**experiment begin**

```

lemma S5: r R ∧ s R ∧ t R → (|=m F7)
  — nitpick[expect=none] — sledgehammer — none — proof
  apply simp — nitpick[expect=none] — sledgehammer — none — proof
  by metis
lemma S4: r R ∧ t R → (|=m F7)
  — nitpick[expect=genuine] — sledgehammer — ctex — no prf
  apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf
  oops
lemma KB4: s R ∧ t R → (|=m F7)

```

— nitpick[expect=none] — sledgehammer — none — proof  
**apply simp** — nitpick[expect=none] — sledgehammer — none — proof  
**by metis**  
**lemma KTB:**  $r R \wedge s R \longrightarrow (\models^m F7)$   
 — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply simp** — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**oops**  
**lemma KT:**  $r R \longrightarrow (\models^m F7)$   
 — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply simp** — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**oops**  
**lemma KB:**  $s R \longrightarrow (\models^m F7)$   
 — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply simp** — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**oops**  
**lemma K4:**  $t R \longrightarrow (\models^m F7)$   
 — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply simp** — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**oops**  
**lemma K:**  $(\models^m F7)$   
 — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply simp** — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**oops**  
**end**

#### experiment begin

**lemma S5:**  $r R \wedge s R \wedge t R \longrightarrow (\models^m F8)$   
 — nitpick[expect=none] — sledgehammer — none — proof  
**apply simp** — nitpick[expect=none] — sledgehammer — none — proof  
**by metis**  
**lemma S4:**  $r R \wedge t R \longrightarrow (\models^m F8)$   
 — nitpick[expect=none] — sledgehammer — none — proof  
**apply simp** — nitpick[expect=none] — sledgehammer — none — proof  
**by metis**  
**lemma KB4:**  $s R \wedge t R \longrightarrow (\models^m F8)$   
 — nitpick[expect=none] — sledgehammer — none — proof  
**apply simp** — nitpick[expect=none] — sledgehammer — none — proof  
**by metis**  
**lemma KTB:**  $r R \wedge s R \longrightarrow (\models^m F8)$   
 — nitpick[expect=none] — sledgehammer — none — proof  
**apply simp** — nitpick[expect=none] — sledgehammer — none — proof  
**by metis**  
**lemma KT:**  $r R \longrightarrow (\models^m F8)$   
 — nitpick[expect=none] — sledgehammer — none — proof  
**apply simp** — nitpick[expect=none] — sledgehammer — none — proof  
**by metis**  
**lemma KB:**  $s R \longrightarrow (\models^m F8)$   
 — nitpick[expect=none] — sledgehammer — none — proof  
**apply simp** — nitpick[expect=none] — sledgehammer — none — proof

**by metis**  
**lemma**  $K4$ :  $t R \longrightarrow (\models^m F8)$   
 — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**oops**  
**lemma**  $K$ :  $(\models^m F8)$   
 — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**oops**  
**end**

**experiment begin**

**lemma**  $S5$ :  $r R \wedge s R \wedge t R \longrightarrow (\models^m F9)$   
 — nitpick[expect=none] — sledgehammer — none — proof  
**apply** *simp* — nitpick[expect=none] — sledgehammer — none — proof  
**by metis**  
**lemma**  $S4$ :  $r R \wedge t R \longrightarrow (\models^m F9)$   
 — nitpick[expect=none] — sledgehammer — none — proof  
**apply** *simp* — nitpick[expect=none] — sledgehammer — none — proof  
**by metis**  
**lemma**  $KB4$ :  $s R \wedge t R \longrightarrow (\models^m F9)$   
 — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**oops**  
**lemma**  $KTB$ :  $r R \wedge s R \longrightarrow (\models^m F9)$   
 — nitpick[expect=none] — sledgehammer — none — proof  
**apply** *simp* — nitpick[expect=none] — sledgehammer — none — proof  
**by metis**  
**lemma**  $KT$ :  $r R \longrightarrow (\models^m F9)$   
 — nitpick[expect=none] — sledgehammer — none — proof  
**apply** *simp* — nitpick[expect=none] — sledgehammer — none — proof  
**by metis**  
**lemma**  $KB$ :  $s R \longrightarrow (\models^m F9)$   
 — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**oops**  
**lemma**  $K4$ :  $t R \longrightarrow (\models^m F9)$   
 — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**oops**  
**lemma**  $K$ :  $(\models^m F9)$   
 — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**apply** *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf  
**oops**  
**end**

**experiment begin**

**lemma**  $S5$ :  $r R \wedge s R \wedge t R \longrightarrow (\models^m F10)$   
 — nitpick[expect=none] — sledgehammer — none — proof

```

apply simp — nitpick[expect=none] — sledgehammer — none — proof
by metis
lemma S4: r R ∧ t R → (|=m F10)
  — nitpick[expect=none] — sledgehammer — none — proof
apply simp — nitpick[expect=none] — sledgehammer — none — proof
by metis
lemma KB4: s R ∧ t R → (|=m F10)
  — nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma KTB: r R ∧ s R → (|=m F10)
  — nitpick[expect=none] — sledgehammer — none — proof
apply simp — nitpick[expect=none] — sledgehammer — none — proof
by metis
lemma KT: r R → (|=m F10)
  — nitpick[expect=none] — sledgehammer — none — proof
apply simp — nitpick[expect=none] — sledgehammer — none — proof
by metis
lemma KB: s R → (|=m F10)
  — nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma K4: t R → (|=m F10)
  — nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma K: (|=m F10)
  — nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply simp — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
end

```

— Summary of experiments: Nitpick: ctex=84 (with simp 42, without simp 42), none=76 (with simp 38, without simp 38), unknwn=0 Sledgehammer: proof=76 (with simp 38, without simp 38), no prf=84 (with simp 42, without simp 42)

— No conflicts  
**end**

## References

- [1] C. Benzmüller. Faithful logic embeddings in HOL — deep and shallow. In C. Barrett and U. Waldmann, editors, *Automated Deduction – CADE-30 – 30th International Conference on Automated Deduction, Stuttgart, Germany, July 28-31, 2025, Proceedings*, volume 15943 of *Lecture Notes in Computer Science*, pages 280–302. Springer, 2025. (preprint: [arXiv:2502.19311](https://arxiv.org/abs/2502.19311)).